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# DESIGN AND ANALYSIS OF QUAD-BAND WILKINSON POWER DIVIDER

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In this paper, the design of equal-split quad-band Wilkinson power divider is presented. The design consists of two quad-band transmission line transformers and four isolation resistors. Standard transmission line theory and even/odd modes analyses are used to obtain closed form expressions that are solved using particle swarm optimization technique to find the required divider parameters (lengths, characteristic impedances, and isolation resistors). Very good matching at all ports and isolation between the output ports are achieved at four arbitrary frequencies. To validate the design approach and the derived equations, a quad-band microstrip line Wilkinson divider is designed, analyzed, and measured.

*Keywords*: Power dividers; transmission line transformer; particle swarm optimization.

## 1. Introduction

Recently, several designs for dual-band and triple-band Wilkinson power dividers have been presented in which different techniques were proposed to design multi-frequency Wilkinson power dividers. A dual-band Wilkinson power divider was proposed in [Wu *et al.*, 2006] which consisted of two-section transmission line transformers (TLT) with a parallel combination of resistor, inductor, and capacitor connecting the output ports. Simple design equations for the divider parameters were derived. Another dual-band Wilkinson power divider was proposed in [Dib & Khodier] in which two isolation resistors were used instead of the parallel RLC combination used in [Wu *et al.*, 2006]. Very recently, a tri-band Wilkinson power divider using three isolation resistors was designed and analyzed in [Chongcheawchamnan *et al.*, 2006]. Numerical optimization was used to find the divider parameters.

In this paper, as an extension of the idea proposed in [Chongcheawchamnan *et al.*, 2006], the design and analysis of quad-band four-section Wilkinson power divider is presented. In this quad-band Wilkinson divider, the quarter-wave sections in the conventional divider are replaced by four-section TLTs. Moreover, four isolation resistors are used. For the even

mode, four non-linear equations are derived, using standard transmission line theory, which are solved using the particle swarm optimization (PSO) technique [Kennedy & Eberhart, 1995; Gies & Rahmat-Samii, 2003; Robinson & Rahmat-Samii, 2004]. This results in the required impedances and lengths of the four-section TLT. Another four non-linear equations are derived using odd-mode analysis, which are also solved using the PSO which gives the values of the isolation resistors.

The PSO method is chosen to solve the problem since; recently, we have been interested in the application of PSO in different electromagnetics and microwave circuits' problems [Khodier & Christodoulou, 2005; Ababneh *et al.*, 2006]. The PSO technique has been successfully applied to antenna design [Gies & Rahmat-Samii, 2003; Robinson & Rahmat-Samii, 2004; Khodier & Christodoulou, 2005], and the results proved that this method is powerful and effective for optimization problems. PSO is similar in some ways to genetic algorithms, but requires less computational bookkeeping and generally fewer lines of code, including the fact that the basic algorithm is very easy to understand and implement. The interested reader can refer to [Gies & Rahmat-Samii, 2003; Robinson & Rahmat-Samii, 2004; Khodier & Christodoulou, 2005; Ababneh *et al.*, 2006], and the references therein, for the details of the PSO algorithm.

## 2. Design

The design for equal-split quad-band Wilkinson power divider (WPD) starts by substituting each quarter-wavelength branch of a conventional WPD by four sections of transmission lines, having characteristic impedances of $Z_1$, $Z_2$, $Z_3$, and $Z_4$, and physical lengths $l_1$, $l_2$, $l_3$ and $l_4$, respectively, as shown in Fig. 1. Then, four isolation resistors are added (one at each end of the four transmission line sections). The even and odd modes analysis is used to determine the characteristic impedances and lengths of the transmission lines sections, and the values of the isolation resistors.

### 2.1. *Even-mode analysis*

In the even-mode analysis, no current flows in the isolation resistors and the WPD can be divided into two quad-band TLTs, as shown in Fig. 2. Thus, the problem reduces to finding the lengths and impedances of the four sections such that a perfect match (between $Z_0$
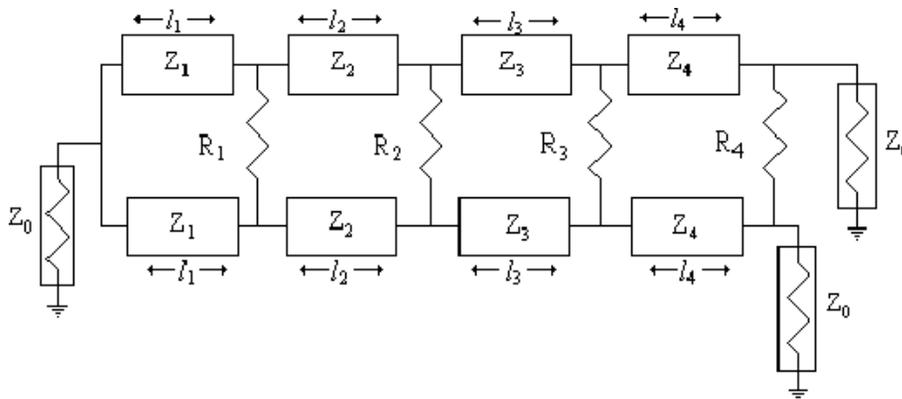


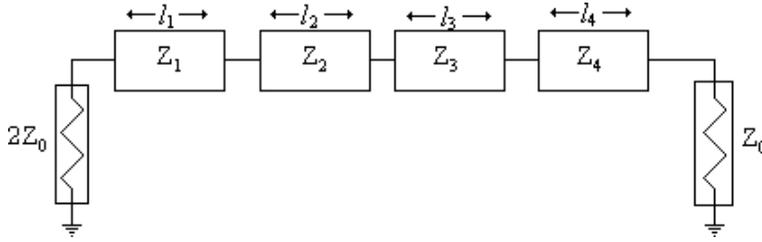Fig. 1. Quad-band WPD with four isolation resistors.

Fig. 2. Even-mode analysis of the quad-band WPD.

and $2Z_0$) is obtained at four arbitrary frequencies $f_1$, $f_2$, $f_3$, and $f_4$. Using transmission line theory and the antimetry condition [Meschanov *et al.*, 1996], the following expression is derived in [Jwaied *et al.*,]:

$$2a + b\frac{\tan(\beta\,\ell_4)}{\tan(\beta\,\ell_3)} + c\frac{\tan(\beta\,\ell_3)}{\tan(\beta\,\ell_4)} + d\tan(\beta\,\ell_4)\tan(\beta\,\ell_3) + \frac{(k-1)}{\tan(\beta\,\ell_4)\tan(\beta\,\ell_3)} = 0, \quad (1)$$

where

$$a = \left(\frac{z_3}{z_4} - \frac{z_4}{z_3}k\right) + \left(\frac{k}{z_3 z_4} - z_3 z_4\right), \tag{2a}$$

$$b = \frac{k}{z_4^2} - z_4^2, \tag{2b}$$

$$c = \frac{k}{z_3^2} - z_3^2, \tag{2c}$$

$$d = \frac{z_4^2}{z_3^2}k - \frac{z_3^2}{z_4^2}. \tag{2d}$$

In (1), normalized impedances are used where $z_4 = Z_4/Z_0$, and $z_3 = Z_3/Z_0$. Moreover, $k$ is the impedance transforming ratio (or the normalized load impedance) which is equal to 2 here. It is clear that there are four unknowns in (1); namely: $z_3$, $z_4$, $l_3$, and $l_4$. Now, (1) should be satisfied at the four design frequencies $f_1$, $f_2$, $f_3$, and $f_4$ which gives four non-linear equations that are solved using the PSO technique. Once $z_3$, $z_4$, $l_3$, and $l_4$ are known, the antimetry conditions [Meschanov *et al.*, 1996] are used to find the other parameters; namely: $l_1 = l_4$, $l_2 = l_3$, and $Z_2 Z_3 = Z_1 Z_4 = 2Z_0^2$.

## 2.2. *Odd-mode analysis*

In this analysis, there is a voltage null along the middle of the circuit shown in Fig. 1. Thus, we can bisect this circuit by grounding the midplane, as shown in Fig. 3. The input admittance seen by one of the output ports is given by:

$$(Z_{in}^{odd})^{-1}$$

$$= 2G_4 + Y_4 \frac{2G_3 + Y_3 \frac{2G_2 + Y_2 \frac{2G_1 + j(Y_2 \tan(\theta_2) - Y_1 \cot(\theta_1)}{Y_2 + Y_1 \cot(\theta_1)\tan(\theta_2) + j2G_1 \tan(\theta_2)} + jY_3 \tan(\theta_3)}{Y_3 + j\tan(\theta_3)\left(2G_2 + Y_2 \frac{2G_1 + j(Y_2 \tan(\theta_2) - Y_1 \cot(\theta_1))}{Y_2 + Y_1 \cot(\theta_1)\tan(\theta_2) + j2G_1 \tan(\theta_2)}\right)} + jY_4 \tan(\theta_4)}{Y_4 + j\tan(\theta_4)\left(2G_3 + Y_3 \frac{2G_2 + Y_2 \frac{2G_1 + j(Y_2 \tan(\theta_2) - Y_1 \cot(\theta_1)}{Y_2 + Y_1 \cot(\theta_1)\tan(\theta_2) + j2G_1 \tan(\theta_2)} + jY_3 \tan(\theta_3)}{Y_3 + j\tan(\theta_3)\left(2G_2 + Y_2 \frac{2G_1 + j(Y_2 \tan(\theta_2) - Y_1 \cot(\theta_1))}{Y_2 + Y_1 \cot(\theta_1)\tan(\theta_2) + j2G_1 \tan(\theta_2)}\right)}\right)},$$
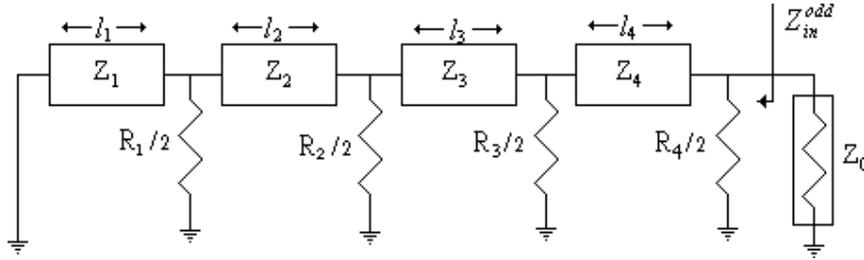
$$\tag{3}$$

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Fig. 3. Odd-mode analysis of the quad-band WPD.

where

$$\theta_1 = \beta\ell_1, \quad \theta_2 = \beta\ell_2, \quad \theta_3 = \beta\ell_3, \quad \theta_4 = \beta\ell_4, \tag{4a}$$

$$G_1 = \frac{1}{R_1}, \quad G_2 = \frac{1}{R_2}, \quad G_3 = \frac{1}{R_3}, \quad G_4 = \frac{1}{R_4}, \tag{4b}$$

$$Y_1 = \frac{1}{Z_1}, \quad Y_2 = \frac{1}{Z_2}, \quad Y_3 = \frac{1}{Z_3}, \quad Y_4 = \frac{1}{Z_4}. \tag{4c}$$

For perfect match at four arbitrary frequencies ($f_1$, $f_2$, $f_3$ and $f_4$), the following equation should be satisfied at these frequencies simultaneously:

$$\mathbf{Z}_{in}^{odd} = Z_0. \tag{5}$$

By imposing (5) at the four frequencies, we get four non-linear equations with four unknowns $G_1$, $G_2$, $G_3$, and $G_4$. The solution of the resulting four non-linear equations is obtained using the PSO method.

Details concerning the PSO algorithm such as the governing equations, parameters, and convergence are thoroughly explained in [Robinson & Rahmat-Samii, 2004; Khodier & Christodoulou, 2005; Ababneh *et al.*, 2006]. The number of particles (or searching agents) used is 25, and the algorithm is stopped once the value of the cost function becomes less than $10^{-15}$. The algorithm is run more than once to make sure that it converges to the same solution each time.

## 3. Results

To validate the above analysis, a design for a quad-band microstrip line WPD is introduced. The terminating impedance is chosen to be $Z_0 = 50\,\Omega$, and the desired operating frequencies are $f_1 = 0.5\,\text{GHz}$, $f_2 = 1\,\text{GHz}$, $f_3 = 1.5\,\text{GHz}$, and $f_4 = 2\,\text{GHz}$. From the analysis part, we get the following values for the design parameters:

$$Z_1 = 86.94\,\Omega, \quad Z_2 = 75.74\,\Omega, \quad Z_3 = 66.02\,\Omega, \quad Z_4 = 57.51\,\Omega,$$

$$l_1 = l_2 = l_3 = l_4 = 36°, \quad \text{where } f_1 \text{ is the reference frequency,}$$

$$R_1 = 117.59\,\Omega, \quad R_2 = 222.94\,\Omega, \quad R_3 = 319.38\,\Omega, \quad R_4 = 422.75\,\Omega.$$

The fabricated WPD (using an FR4 substrate) is shown in Fig. 4, which has an overall size of $12 \times 5$ cm. For practical reasons, $R_1$ was chosen to be $120\,\Omega$, $R_2 = 220\,\Omega$, $R_3 = 330\,\Omega$, and $R_4 = 423\,\Omega$.

Figure 5 presents the simulated performance of the quad-band WPD obtained using the software Ansoft Designer SV [www.ansoft.com] in the frequency range 0–2.5 GHz. It can
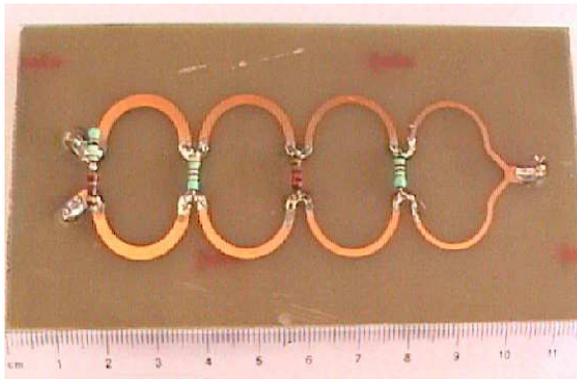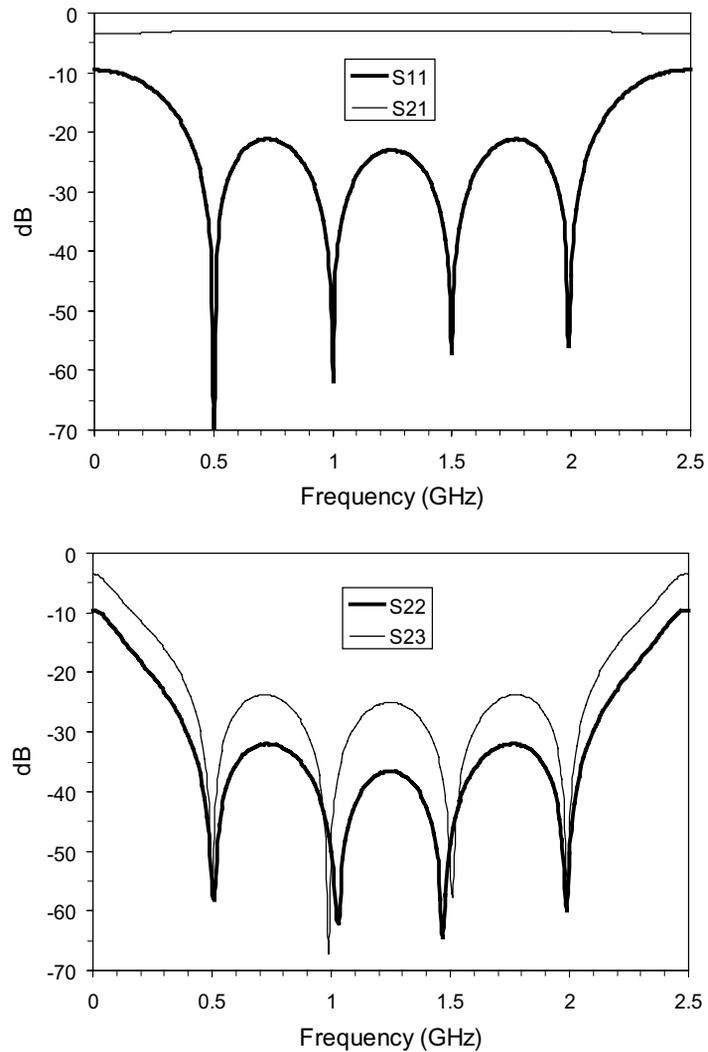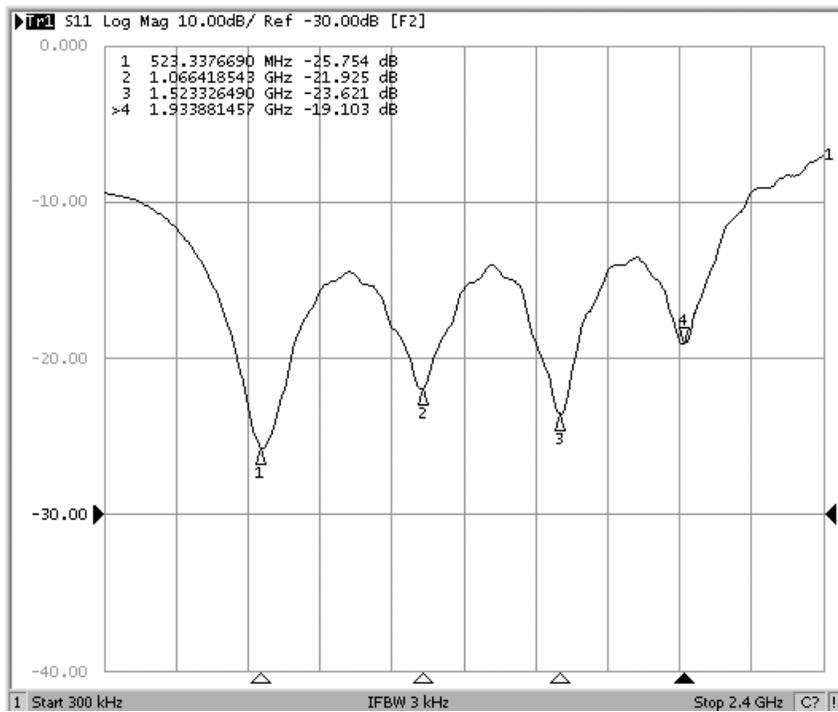
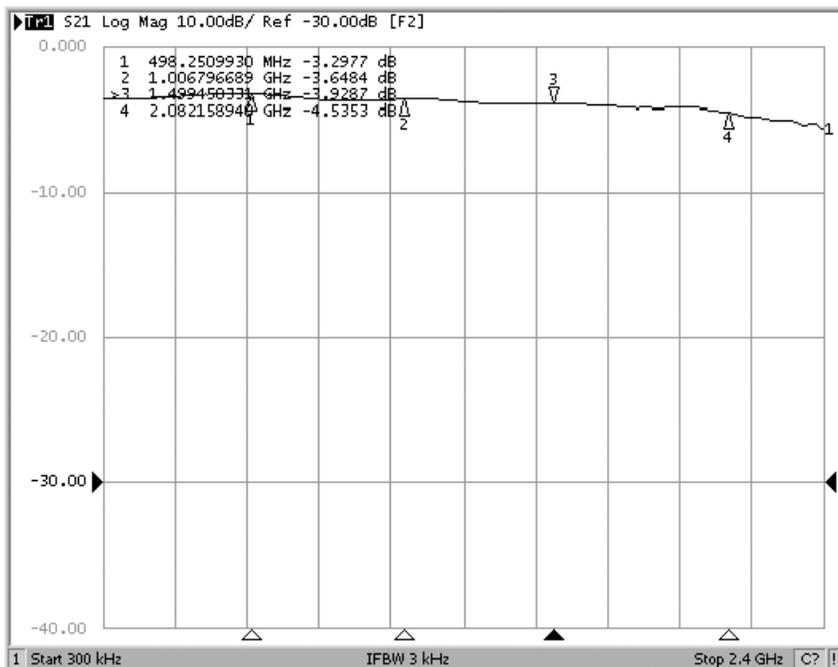Fig. 4. Photograph of the fabricated quad-band WPD.





Fig. 5. Simulated $S$-parameters for the quad-band WPD with $R_1 = 120\,\Omega$, $R_2 = 220\,\Omega$, $R_3 = 330\,\Omega$, and $R_4 = 423\,\Omega$.

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(a) $S_{11}$



(b) $S_{21}$ (or $S_{31}$).

Fig. 6. Measured $S$-parameters for the microstrip line quad-band WPD.
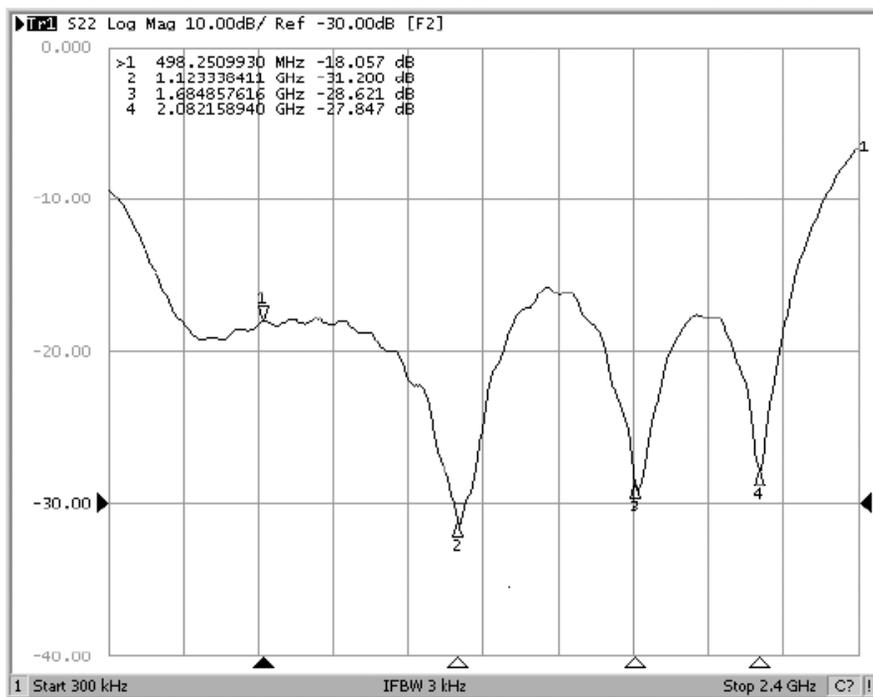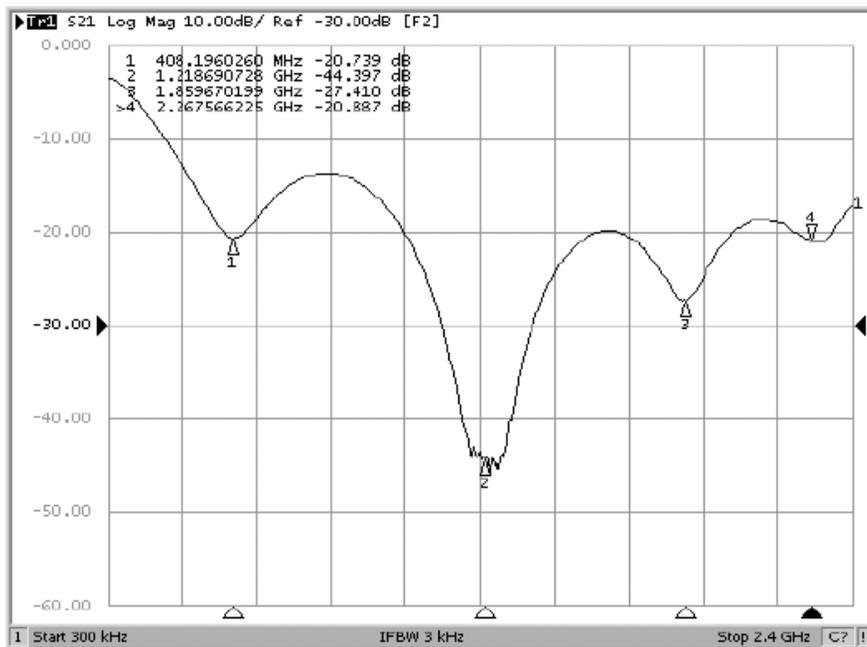
(c) $S_{22}$ (or $S_{33}$).



(d) $S_{23}$ (isolation between the output ports two and three).

Fig. 6. (*Continued*)

be seen that very good match at the three ports is obtained at the four design frequencies. Return loss at the three ports of less than 10 dB is obtained in the whole frequency range. Moreover, very good isolation between the two output ports is obtained at these frequencies. The equal-split condition can be clearly observed too. Figure 6 presents the measured *S*-parameters for this microstrip quad-band WPD in the frequency range 300 KHz–2.4 GHz. Matching at the input port, isolation between the output ports, and the equal-split properties can be clearly seen around the four design frequencies. The discrepancies between the measured and simulated response could be due to the connectors, measurement inaccuracy, and the use of carbon resistors instead of surface mount resistors.

## 4.  Conclusions

As an application of the quad-band four-section transmission line impedance transformer, a design for quad-band four-section WPD with four isolation resistors is presented. Very good matching and isolation were achieved at four arbitrary frequencies. Simulated and experimental results for a microstrip line WPD were provided to validate the design.

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