Confidence Intervals to Assess Variation in Fat Content at a Fast-Food Restaurant

Hamburrger, Inc. is a fast-food restaurant serving hamburgers among a few other items. The restaurant claims that the average fat content in the hamburgers is 15 grams.

Section 3.1 gives a brief description of the define phase. Section 3.2 illustrates the measure phase with detailed instructions for using Minitab®. The analyze phase is briefly discussed in Section 3.3. Section 3.4 illustrates the improve phase with detailed instructions for using Minitab®. Finally, the control phase is briefly discussed in Section 3.5.

3.1 Define Phase

A few of the customers complained to the operations manager recently that the fat content in the hamburgers appeared to be higher than the restaurant’s claim of 15 grams. The operations manager wishes to use 95% confidence intervals to verify whether the restaurant’s claim of 15 grams (average) of fat in the hamburgers is correct. He also wishes to verify his assumption that the standard deviation of the fat content is less than 1 gram.

3.2 Measure Phase

The operations manager randomly selects 20 hamburgers and measures the fat content (in grams) in each of them as follows: 15.5, 12.3, 15.4, 16.5, 15.9, 17.1, 16.9, 14.3, 19.1, 18.2, 18.5, 16.3, 20.0, 19.0, 15.6, 13.5, 14.0, 16.5, 19.0, and 18.6.

Open the CHAPTER_3_BEFORE.MTW worksheet that has the above data (the worksheet is available at the publisher’s website; the data from the worksheet are also provided in the Appendix). Before constructing a confidence interval for the above data, it is important to check whether the data are in statistical control. Because each number is for one hamburger, the appropriate set of control charts is I-MR (individual and moving range) charts. Figure 3.1 shows how to select I-MR charts in Minitab®. Doing so will open
the dialog box shown in Figure 3.2. Select “Fat” for “Variable”, and click on “OK.” The I-MR charts shown in Figure 3.3 will be the result. As is evident, the data are in statistical process control.

Now that the data are in statistical control, the operations manager wishes to construct the 95% confidence interval for the mean of fat content in
population of the hamburgers. He also wishes to check whether the data are normally distributed before relying on the confidence interval. The “Graphical Summary” option in Minitab® produces both the normality test result as well as the confidence interval. Figure 3.4 shows how to select the graphical summary. Doing so will open the dialog box shown in Figure 3.5. Select “Fat” for “Variable” and click on “OK”. The graphical summary shown in Figure 3.6 will be the result. Because the $P$-value (0.624) of the Anderson–Darling normality test is greater than 0.05, the data are normally distributed. Notice that the population mean is estimated to be between 15.625 grams and 17.595 grams. The entire confidence interval (15.625, 17.595) is greater than the claim of 15 grams, therefore the operations manager considers this a serious issue.

The operations manager then proceeds to verify his assumption that the standard deviation of the fat content is less than 1 gram. Figure 3.7 shows how to select “1 Variance” in Minitab®. Doing so will open the dialog box shown in Figure 3.8. Select “Samples in columns” for “Data” and “Fat” for “columns”. Click on “Options” and it opens the dialog box shown in Figure 3.9. Select “less than” for “Alternative” because the operations manager is interested in the standard deviation being less than 1 gram. Click on “OK” and it takes you back to the dialog box shown in Figure 3.8. Click on “OK” and the output shown in Figure 3.10 is the result. Because the data are normally distributed, we must look at the upper bound (2.89 grams) of the standard deviation given by the chi-square method. This means that 95% of
### FIGURE 3.4
Selection of “Graphical Summary” before improvement.

![Graphical Summary Window]

### FIGURE 3.5
Variable of “Fat” for graphical summary before improvement.

![Variable Selection]

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Confidence Intervals to Assess Variation in Fat Content

FIGURE 3.6
Graphical summary before improvement.

FIGURE 3.7
Selection of “1 Variance” before improvement.

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the hamburgers have fat content with a standard deviation that is lower than 2.89 grams. Inasmuch as 2.89 grams is much greater than the assumption of 1 gram, the operations manager considers this also a serious issue.

Before proceeding to the improve phase, the operations manager wishes to check how many of the customers are dissatisfied. To this end, he randomly selects 1,000 customers who bought hamburgers and finds that 83 of them are dissatisfied with the food quality. He then proceeds to construct a confidence interval for the proportion of all customers who are dissatisfied. Figure 3.11 shows how to select “1 Proportion” in Minitab®. Doing so will open the dialog box shown in Figure 3.12. Enter “83” for “Number of events” and “1000” for “Number of trials”. Click on “Options” and the dialog box shown in Figure 3.13 opens. Select “less than” for “Alternative” because the operations manager is interested in lowering the proportion of all customers who are dissatisfied. Also, check the box for “Use test and interval based on normal distribution”. Click on “OK” and it takes you back to the dialog box shown in Figure 3.12. Click on “OK” and the output shown in Figure 3.14 is
Test and CI for One Variance: Fat

Method
The chi-square method is only for the normal distribution. The Bonett method is for any continuous distribution.

Statistics
<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>StDev</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>20</td>
<td>2.11</td>
<td>4.43</td>
</tr>
</tbody>
</table>

95% One-Sided Confidence Intervals

<table>
<thead>
<tr>
<th>Variable</th>
<th>Method</th>
<th>StDev</th>
<th>for Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>Chi-Square</td>
<td>2.89</td>
<td>8.33</td>
</tr>
<tr>
<td></td>
<td>Bonett</td>
<td>2.81</td>
<td>7.92</td>
</tr>
</tbody>
</table>

FIGURE 3.10
One-sided 95% confidence interval for standard deviation before improvement.

FIGURE 3.11
Selection of “1 Proportion” before improvement.

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FIGURE 3.12
Entry of “Number of events” and “trials” before improvement.

FIGURE 3.13
Selection of “less than” option for estimating population proportion before improvement.

Test and CI for One Proportion

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83</td>
<td>1000</td>
<td>0.083000</td>
<td>0.097350</td>
</tr>
</tbody>
</table>

Using the normal approximation.

FIGURE 3.14
One-sided 95% confidence interval for population proportion before improvement.
Confidence Intervals to Assess Variation in Fat Content

the result. It is clear that there is a 95% probability that the proportion of all customers who are dissatisfied is less than 0.097 (or 9.7%). The operations manager wishes to lower this upper bound estimate.

3.3 Analyze Phase

The operations manager analyzes the process and discovers that the fat content and its variation are highly affected by the amount of oil used by the employees on the three different grills used to make hamburgers.

3.4 Improve Phase

The operations manager standardizes the process so that the use of oil on the three different grills is controlled. An automatic machine is bought and installed to dispense oil on a grill each time a hamburger is made.

In order to check whether the process has really improved, the operations manager randomly selects 20 hamburgers and measures the fat content (in grams) in each of them as follows: 14.9, 15.0, 15.4, 15.3, 15.2, 15.1, 14.9, 14.8, 15.6, 14.5, 15.3, 15.8, 15.0, 15.0, 14.3, 15.3, 15.2, 14.7, 15.1, and 14.7.

Open the CHAPTER_3_AFTER.MTW worksheet that has the above data (the worksheet is available at the publisher’s website; the data from the worksheet are also provided in the Appendix). Before constructing a confidence interval for the above data, it is important to check whether the data are in statistical control. Because each number is for one hamburger, the appropriate set of control charts is I-MR (individual moving range) charts. The I-MR charts for the above data are shown in Figure 3.15. As is evident, the data are in statistical process control.

Now that the data are in statistical control, the operations manager wishes to construct the 95% confidence interval for the mean of fat content in the population of hamburgers. He also wishes to check whether the data are normally distributed before relying on the confidence interval. Figure 3.16 shows the graphical summary. Because the P-value (0.925) of the Anderson–Darling normality test is greater than 0.05, the data are normally distributed. Notice that the population mean is estimated to be between 14.887 grams and 15.223 grams. Inasmuch as the claim of 15 grams is within the interval (14.887, 15.223), it is clear that the process has improved with respect to the mean fat content.

The operations manager then proceeds to verify his assumption that the standard deviation of the fat content is less than 1 gram. As is clear from the
FIGURE 3.15
I-MR charts after improvement.

FIGURE 3.16
Graphical summary after improvement.

<table>
<thead>
<tr>
<th>Anderson-Darling Normality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-Squared</td>
</tr>
<tr>
<td>P-Value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary for Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td><strong>StDev</strong></td>
</tr>
<tr>
<td><strong>Variance</strong></td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
</tr>
<tr>
<td><strong>N</strong></td>
</tr>
</tbody>
</table>

| Minimum         | 14.300 |
| 1st Quartile    | 14.825 |
| Median          | 15.050 |
| 3rd Quartile    | 15.300 |
| Maximum         | 15.800 |

95% Confidence Interval for Mean: 14.887 - 15.223
95% Confidence Interval for Median: 14.900 - 15.276
95% Confidence Interval for StDev: 0.273 - 0.524
Confidence Intervals to Assess Variation in Fat Content

The output shown in Figure 3.17, the upper bound is 0.492 gram for the standard deviation (given by the chi-square method). This means that 95% of the hamburgers have fat content with a standard deviation that is lower than 0.492 gram. Because 0.492 gram is less than the assumption of 1 gram, it is evident that the process has improved with respect to the standard deviation as well.

One can also simulate confidence intervals using Minitab®. The operations manager wishes to perform the simulation for 10 normally distributed samples with a sample size of 20 hamburgers. Doing what is shown in Figure 3.18 opens the dialog box shown in Figure 3.19. Select “20” for “Number of rows of data to generate”. Enter “C3-C12” for “Store in column(s)”. Because, after process improvement, the population mean is approximately 15 grams and the population standard deviation is approximately 0.5 grams, enter “15” for “mean” and “0.5” for “Standard deviation”. Click on “OK” and the data shown in Figure 3.20 are the result.

Before proceeding to simulation, it is a good idea to check whether the data generated are in statistical control. The sample size is greater than 10, therefore the appropriate control charts are Xbar-S charts (sample means and sample standard deviations). Figure 3.21 shows how to select “Xbar-S”. Doing so will open the dialog box shown in Figure 3.22. Select “Observations for a subgroup are in one row of columns” from the drop-down menu, and select “C3-C12” in the empty box below the menu. Click on “OK” and the Xbar-S charts shown in Figure 3.23 are the result. It is clear that the data are in statistical control.

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FIGURE 3.18
Selection of “Normal” to generate normally distributed data.

FIGURE 3.19
Mean and standard deviation for generation of random samples.
For simulating the confidence intervals for the 10 samples generated, select “Interval Plot” as shown in Figure 3.24. Doing so opens the dialog box shown in Figure 3.25. Select “Simple” under “Multiple Ys” and click on “OK”. That will open the dialog box shown in Figure 3.26. Select “C3-C12” columns for “Graph variables” and click on “OK”. The intervals plot (95% confidence intervals) for the 10 samples shown in Figure 3.27 are created. As shown in Figure 3.28, right-click anywhere on the interval plot and select “Reference Lines”. This opens the dialog box shown in Figure 3.29. Enter “15” for “Show reference lines at Y values” and click on “OK”. This adds the reference line of 15, as shown in Figure 3.30. Because most of the confidence intervals contain the population mean of 15 grams, this simulation confirms that the population mean estimate is indeed 15 grams. If the simulation is performed for a large number of samples, 95% of the confidence intervals are expected to contain the population mean of 15 grams.

The operations manager now wishes to check how many of the customers are dissatisfied after the process has improved. To this end, he randomly selects 1,200 customers who bought hamburgers and finds that 25 of them are dissatisfied with the food quality. He then proceeds to construct a confidence interval for the proportion of all customers who are dissatisfied. In the dialog box shown in Figure 3.31, enter “25” for “Number of events” and
FIGURE 3.21
Selection of “Xbar-S”.

FIGURE 3.22
Selection of variables for Xbar-S charts.
Confidence Intervals to Assess Variation in Fat Content

FIGURE 3.23
Xbar-S charts.

FIGURE 3.24
Selection of "Interval Plot" for simulation of confidence intervals.

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FIGURE 3.25
Selection of “Simple” interval plots under “Multiple Y’s”.

FIGURE 3.26
Selection of variables for simulation.
Confidence Intervals to Assess Variation in Fat Content

Interval Plot of C3, C4, C5, C6, C7, C8, C9, C10, C11, C12
95% CI for the Mean

FIGURE 3.27
Interval plot without reference line.

FIGURE 3.28
Process to show reference line.

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**FIGURE 3.29**
Entry of reference line.

**FIGURE 3.30**
Interval plot with reference line.
“1200” for “Number of trials”. Click on “Options” and select “less than” for “Alternative” because the operations manager is interested in lowering the proportion of all customers who are dissatisfied. Also, check the box for “Use test and interval based on normal distribution”. Click on “OK” and it takes you back to the dialog box shown in Figure 3.31. Click on “OK” and the output shown in Figure 3.32 is the result. It is clear that there is a 95% probability that the proportion of all customers who are dissatisfied is less than 0.027 (or 2.7%). The operations manager is satisfied that the process has improved.
3.5 Control Phase

The operations manager plans to check the hamburger preparation process on the three grills regularly, in order to ensure that the customers consistently get what they are promised.