The F-distribution
Let \( X_1, X_2, \ldots, X_n \) be a random sample from a normal distribution with mean \( \mu \) and variance \( \sigma^2 \), and let \( S^2 \) be the sample variance. Then the random variable

\[
X^2 = \frac{(n - 1) S^2}{\sigma^2}
\]

has a chi-square (\( \chi^2 \)) distribution with \( n - 1 \) degrees of freedom.
$\chi^2$ Distribution

Graph showing the $\chi^2_k$ distribution for different values of $k$: $k=1, k=2, k=3, k=4, k=6, k=9$. The graph plots $f_k(x)$ against $x$ for each value of $k$. The curves represent the probability density functions for each degree of freedom.
The *F* Distribution

Let *W* and *Y* be independent chi-square random variables with *u* and *v* degrees of freedom respectively. Then the ratio

\[ F = \frac{W/\nu}{Y/\nu} \]

has the probability density function

\[
f(x) = \frac{\Gamma\left(\frac{u + v}{2}\right) \left(\frac{u}{v}\right)^{u/2} x^{(u/2) - 1}}{\Gamma\left(\frac{u}{2}\right) \Gamma\left(\frac{v}{2}\right) \left[\left(\frac{u}{v}\right)x + 1\right]^{(u+v)/2}}, \quad 0 < x < \infty
\]

and is said to follow the distribution with *u* degrees of freedom in the numerator and *v* degrees of freedom in the denominator. It is usually abbreviated as *F*<sub>*u,v*</sub>.
The $F$ Distribution
The *F* Distribution

Let \( X_{11}, X_{12}, \ldots, X_{1n_1} \) be a random sample from a normal population with mean \( \mu_1 \) and variance \( \sigma^2_1 \), and let \( X_{21}, X_{22}, \ldots, X_{2n_2} \) be a random sample from a second normal population with mean \( \mu_2 \) and variance \( \sigma^2_2 \). Assume that both normal populations are independent. Let \( s_1^2 \) and \( s_2^2 \) be the sample variances. Then the ratio

\[
F = \frac{s_1^2 / \sigma^2_1}{s_2^2 / \sigma^2_2}
\]

has an *F* distribution with \( n_1 - 1 \) numerator degrees of freedom and \( n_2 - 1 \) denominator degrees of freedom.
The $F$ Distribution

Null hypothesis: \[ H_0 : \sigma_1^2 = \sigma_2^2 \]

Test statistic: \[ F_0 = \frac{S_1^2}{S_2^2} \quad \text{(10-31)} \]

<table>
<thead>
<tr>
<th>Alternative Hypotheses</th>
<th>Rejection Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1 : \sigma_1^2 \neq \sigma_2^2$</td>
<td>$f_0 &gt; f_{\alpha/2, n_1-1, n_2-1}$ or $f_0 &lt; f_{1-\alpha/2, n_1-1, n_2-1}$</td>
</tr>
<tr>
<td>$H_1 : \sigma_1^2 &gt; \sigma_2^2$</td>
<td>$f_0 &gt; f_{\alpha, n_1-1, n_2-1}$</td>
</tr>
<tr>
<td>$H_1 : \sigma_1^2 &lt; \sigma_2^2$</td>
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</tr>
</tbody>
</table>

\[ f_{\alpha/2, n_1-1, n_2-1} = \frac{1}{f_{1-\alpha/2, n_2-1, n_1-1}} \]