



### SEISMIC DEMANDS ASSESSMENT OF TALL BUILDINGS

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**Keywords:** Inelastic response, spectra, ductility, tall building.

**Abstract.** *Currently, in order to estimate the seismic demand of a nonlinear structure, some methods always require a repeatedly iterative procedure no matter the elastic or inelastic response spectra was used in the procedure. Many studies dealt with the development of different inelastic spectra with the aim to simplify the evaluation of inelastic deformations and performance of structures. Recently, the concept of inelastic spectra has been adopted in the global scheme of the performance-based seismic design through capacity-spectrum methods. In this paper, an improved procedure applicable to the analysis and design of tall buildings is presented and illustrated by examples. Also, it is a new procedure for estimating the seismic deformation of Multi-Degree-of-Freedom systems. The accuracy of the improved procedure is verified against the nonlinear time history analysis results of a 9-story SAC steel building. The comparison showed that the new method is capable to furnish accurate deformations and responses.*

## 1 INTRODUCTION

The seismic demands assessment methods are generally based on the nonlinear static analysis, where the structure is subjected to loads increasing monotonically with a constant load distribution over the entire height to until a predetermined target displacement. The distribution of these forces and the target displacement are based on the assumption that the response is controlled only by the fundamental mode where the mode shape remains unchanged in the inelastic domain. Knowing that constant distribution of forces will not capture the contribution of higher modes in the overall structural response, several researchers have proposed adaptive force distributions that attempt to follow more closely the distribution of inertial forces over time [11, 3 and 13]. These adaptive schemes are conceptually complicated and involve complicated numerical analysis techniques. Attempts have also been made to consider more than the fundamental mode of vibration in the Pushover analysis [19, 21, 13, 18, 7 and 5].

The Modal Pushover Analysis (MPA), recently developed by Chopra and Goel (2001) [7], which includes the effects of higher modes of vibration, provides a good estimation of the seismic demand while maintaining a simplistic concept and computing attraction due to the invariability of the distribution of forces. After, the modal pushover



analysis undergoes changes by Chopra et al (2004) [9], for which an extension of this analysis was developed under the name (Modified Modal Pushover Analysis, MMPA) that combines the elastic influence of higher modes with inelastic response of the first mode using several combinations such as the Square Roof of the Sum of Squares (SRSS). Unlike the MMPA, the Pushover analysis of upper limits (Upper-Bound Pushover Analysis, UBPA) (Jan et al. 2004) [14], is based on the use of a load vector obtained by combining the vector of the first mode and the second corrected vector mode.

Recently, Pushover analysis has undergone modifications to be applicable on buildings asymmetric under bi-directional excitations [16, 12, 2 and 17]. Finally, Pushover analysis has also been extended to general three-dimensional torsion coupled systems in several studies as [20, 4 and 15].

In this paper, a new method is developed to evaluate the evaluation of seismic demands of MDOF systems. This method is called in this paper the Approximate Multimodal Dynamic Analysis. This procedure, is developed to consider the contributions of higher modes of vibration for tall buildings.

## 2 APPROXIMATE MULTIMODAL DYNAMIC ANALYSIS

The matrix form of differential equations governing the response of a MDOF system to earthquake induced ground motion can be written as:

$$M\ddot{x}(t) + C\dot{x}(t) + F(x, \text{sign}\dot{x}) = -M\iota\ddot{x}_g(t) \quad (1)$$

Where  $M$  and  $C$  are the mass and damping matrices respectively,  $F$  denotes the resisting force vector,  $\iota$  is the vector of earthquake influence coefficients and  $\ddot{x}_g(t)$  denotes the earthquake acceleration. The damping matrix  $C$  would not be needed in this analysis of earthquake response; instead modal damping ratios suffice.

The resisting force vector  $F$  is defined as the sum of the linear and the hysteretic parts as represented in Fig. 1.

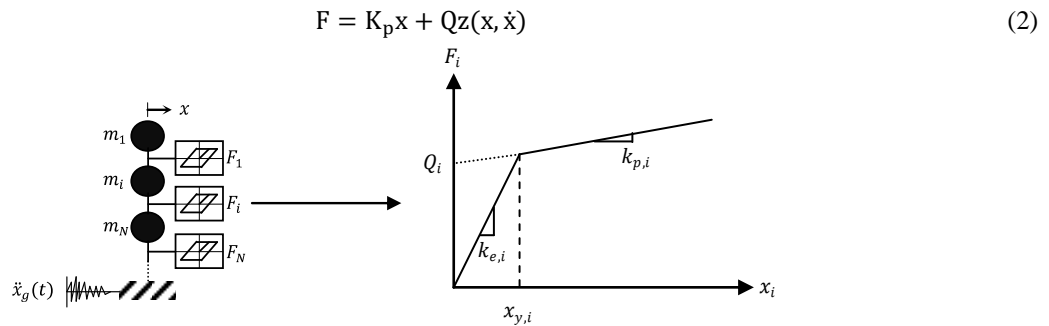


Figure. 1 Example of the resisting force of a MDOF system

Where,  $m_i$ ,  $F_i$ ,  $k_{e,i}$ ,  $k_{p,i}$ ,  $Q_i$  and  $x_{y,i}$  are the mass, resisting force, elastic stiffness, postyield stiffness, yield strength and yield displacement of the  $i$ th level, respectively.

In Eq. (2), the resisting force is a vector for MDOF systems,  $K_p$  is the postyield stiffness matrix,  $Q$  the yield strength vector, and  $z$  a dimensionless variable that characterizes the Bouc-Wen model of hysteresis [22]. It is given by:

$$\dot{z} = \frac{\dot{x}}{x_y} [A - |z|^\lambda (B \text{sign}(\dot{x}z) + \beta)] \quad (3)$$



Where,  $x_y$  is the yield displacement vector;  $A, B, \lambda$  and  $\beta$  are the parameters that control the shape of the hysteresis loop which are taken as:  $A = 1$ ,  $B = 0.1$ ,  $\lambda = 0.9$  and  $\beta = 6$  for bilinear system,  $\text{sign}(\cdot)$  is the sign function [22].

Using Eq. (1) and Eq. (2) we get [5]:

$$M\ddot{x}(t) + C\dot{x}(t) + K_p x(t) + Qz(x, \dot{x}) = -M\ddot{x}_g(t) \quad (4)$$

The decomposition of the MDOF system as a series of normal modes is reasonable. Eq. (5) is used to involve the influence of higher modes in the peak and overall response of the structure [10].

$$x(t) = \sum_n x_n(t) = \sum_n \phi_n \gamma_n(t) \quad (5)$$

Where:  $\gamma_n(t)$  is the modal coordinate and  $\phi_n$  is the  $n$ th natural vibration mode of the structure.

Substituting Eq. (4) into Eq. (5), using the mass, stiffness and classical damping orthogonality mode properties, we obtain the following differential equation for the single-degree-of-freedom (SDOF) system response:

$$\ddot{\gamma}_n(t) + 2\xi_n \omega_n \dot{\gamma}_n(t) + \alpha_n \omega_n^2 \gamma(t) + \frac{Q_n z_n(\gamma, \dot{\gamma})}{M_n^*} = -\Gamma_n \ddot{x}_g(t) \quad (6)$$

Where,  $\omega_n$  is the natural vibration frequency,  $\xi_n$  the damping ratio,  $\alpha_n$  the post-to-preyield stiffness ratio,  $Q_n = \phi_n^T Q$  the yield strength,  $M_n^* = \frac{L_n}{\Gamma_n}$ , the effective mass,  $\Gamma_n = \phi_n^T m / \phi_n^T m \phi_n$  the modal participation factor and  $L_n = \phi_n^T m$  for the  $n$ th natural vibration mode.

The solution,  $\gamma_n$ , of Eq. (6) is given by [10]:

$$\gamma_n(t) = \Gamma_n D_n(t) \quad (7)$$

With this approximation, the solution of Eq. (6) can be expressed by Eq. (7), where the displacement  $D_n(t)$  of the SDOF system can be assessed by the following equation [5]:

$$\ddot{D}_n(t) + 2\xi_n \omega_n \dot{D}_n(t) + \alpha_n \omega_n^2 D(t) + \frac{Q_n z_n(D, \dot{D})}{\Gamma_n M_n^*} = -\ddot{x}_g(t) \quad (8)$$

This ductility demand (or ductility factor) for the SDOF bilinear system is expressed as:

$$\mu_n = \frac{D_{n,m}}{D_{n,y}} \quad (9)$$

Where:  $D_{n,m}$  is the peak displacement and  $D_{n,y}$  is the yield displacement.

It seems worth to associate for each instantaneous inelastic displacement  $D_n(t)$  an instantaneous ductility factor  $\mu_n(t)$  defined as:

$$\begin{cases} D_n(t) = \mu_n(t) \times D_{n,y} \\ \dot{D}_n(t) = \dot{\mu}_n(t) \times D_{n,y} \\ \ddot{D}_n(t) = \ddot{\mu}_n(t) \times D_{n,y} \end{cases} \quad (10)$$

Eq. (8) can be rewritten in terms of ductility factor  $\mu_n$ , by substituting Eq. (10) in Eq. (8) and dividing by  $D_{n,y}$ , which gives:



$$\ddot{\mu}_n + 2\xi_n\omega_n\dot{\mu}_n + \alpha_n\omega_n^2\mu_n + \frac{q_n g z_n(\mu, \dot{\mu})}{D_{n,y}} = -\frac{1}{D_{n,y}}\ddot{x}_g(t) \quad (11)$$

$q_n$  is the yield strength coefficient for the  $n$ th natural vibration mode of the structure (defined as yield strength divided by  $L_n$ ).

$$q_n = \frac{Q_n}{L_n} \quad (12)$$

Also, Eq. (3) may be expressed in terms of ductility factor  $\mu_n$  as:

$$\dot{z} = \dot{\mu}_n [A - |z|^\lambda (B \text{sign}(\dot{x}z) + \beta)] \quad (13)$$

The term  $\frac{q_n g}{D_{n,y}}$  in Eq. (11) is rewritten as [1]:

$$\frac{q_n g}{D_{n,y}} = \omega_n^2 (1 - \alpha_n) \quad (14)$$

Substituting Eq. (14) into Eq. (11) gives:

$$\ddot{\mu}_n + 2\xi_n\omega_n\dot{\mu}_n + \alpha_n\omega_n^2\mu_n + \omega_n^2(1 - \alpha_n)z_n(\mu, \dot{\mu}) = -\frac{\omega_n^2(1 - \alpha_n)}{q_n g}\ddot{x}_g(t) \quad (15)$$

It can be observed from Eq. (15) that for a given ground acceleration,  $\mu_n(t)$  depends on  $\xi_n$ ,  $\omega_n$ ,  $\alpha_n$  and  $q_n$  of the  $n$ th natural vibration mode.

Based on Eq. (5) and (7) and dividing by  $D_{n,y}$  give the ductility demand and the displacement of the original structure [5]:

$$\mu(t) = \sum_n \phi_n \Gamma_n \mu_n(t) \quad x(t) = \sum_n \phi_n \Gamma_n D_n(t) \quad (16)$$

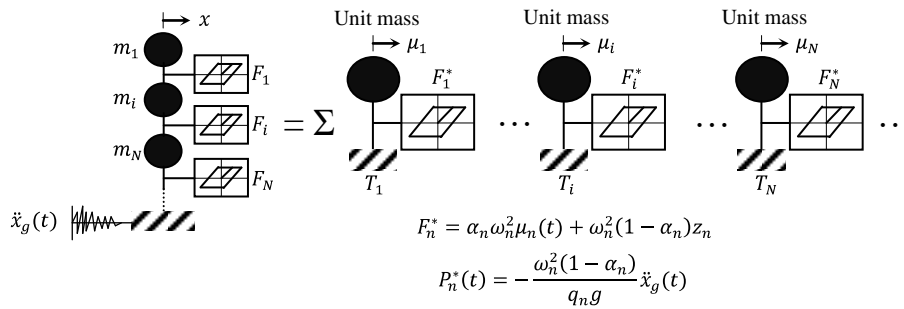


Figure. 2 Approximate multimodal dynamic procedure for MDOF structures

Fig. 2 illustrates the technique of uncoupling the equation of motion in terms of ductility factor characterizing the MDOF system. The response of a MDOF system to earthquake ground motion can be computed as a function of time by the procedure just developed the approximate multimodal dynamic analysis (AMDA), which is detailed in the next application. The proposed approximate analysis consists to solve Eq. (15) for  $\ddot{x}_g(t)$  that will be multiplied by a new factor  $-\omega_n^2(1 - \alpha_n)/q_n g$  to constitute a new excitation for the structure to determine finally the total response



quantities of interest by using Eq. (16).

### 3 APPLICATION

In recent years Chopra and Goel (2002) assessed the strength variation of several procedures including the modal Pushover analysis (MPA), that they developed. The MPA analysis is based on structural dynamics theory. Its accuracy and reliability in estimating the peak response of inelastic MDOF systems has been evaluated extensively by the authors. Chopra and Goel (2004) analyzed and evaluated the response of several procedures for nonlinear static analysis, including Pushover analysis where only fundamental mode was taken into account.

The accurate of the proposed procedure is evaluated for a 9-story SAC steel building [7]. The 'exact' response of a rigorous nonlinear time history analysis (NL-THA) is compared with the response obtained by the approximate multimodal dynamic analysis (AMDA).

The 9-story structure meets the seismic code requirements and represents typical medium-rise buildings designed for the Los Angeles, California region. The Pushover curves of this structure presented in [7] are sufficient for the objectives of this study. The selected structure is tested and detailed in this section when subjected to one time and half (1.5) El Centro 1940 ground motion. The properties of the first three modes of vibration are summarized in Table 1.

The capacity curves of the three first modes are shown in Fig. 3. Next, the Pushover curves are transformed to equivalent SDOF systems (see Fig. 3). The conversion of the idealized Pushover curve to the force-displacement, (see Fig. 3(b)) for the  $n$ th-mode of inelastic SDOF system is obtained by using  $(F_n^* - D_{n,y})$  :

$$S_{an} = \frac{V_{bn}}{M_n^*} = F_n^*, \quad D_n = \frac{x_{rn}}{\Gamma_n \phi_{rn}} \quad (17)$$

Properties	Mode 1	Mode 2	Mode 3
$L_n (kg)$	2736789	-920860	696400
$\Gamma_n$	1.36	-0.5309	0.2406
$M_n^* (kg)$	3740189	488839.1	167531.5
$D_{n,y} (cm)$	26.51	18.65	19.12
$T_n (sec)$	2.2671	0.8525	0.4927
$\alpha_n$	0.19	0.13	0.14
$k_n (kN/cm)$	210.3867	500.2020	1132.6086
$\xi_n (\%)$	1.948	1.103	1.136
$Q_n (kN)$	6168.977	4374.343	4414.347
$q_n (g)$	0.168	0.912	2.685

Table 1: Properties of modal inelastic SDOF systems

In which  $S_{an}$  is the spectral acceleration,  $V_{bn}$  the base shear,  $\phi_{rn}$  is the amplitude of  $\phi_n$  and  $x_{rn}$  the roof displacement.



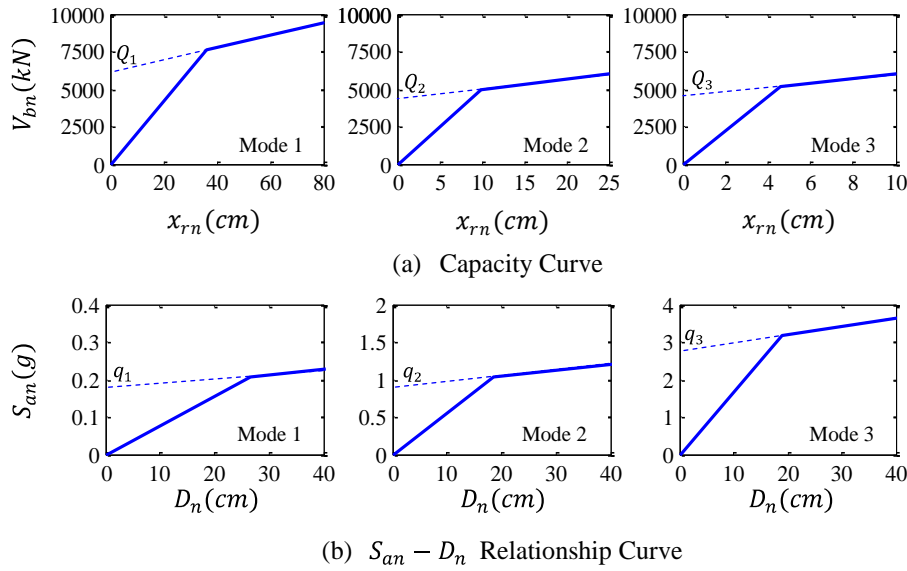


Figure. 3 Modal Pushover curves and capacity diagrams for the first three modes

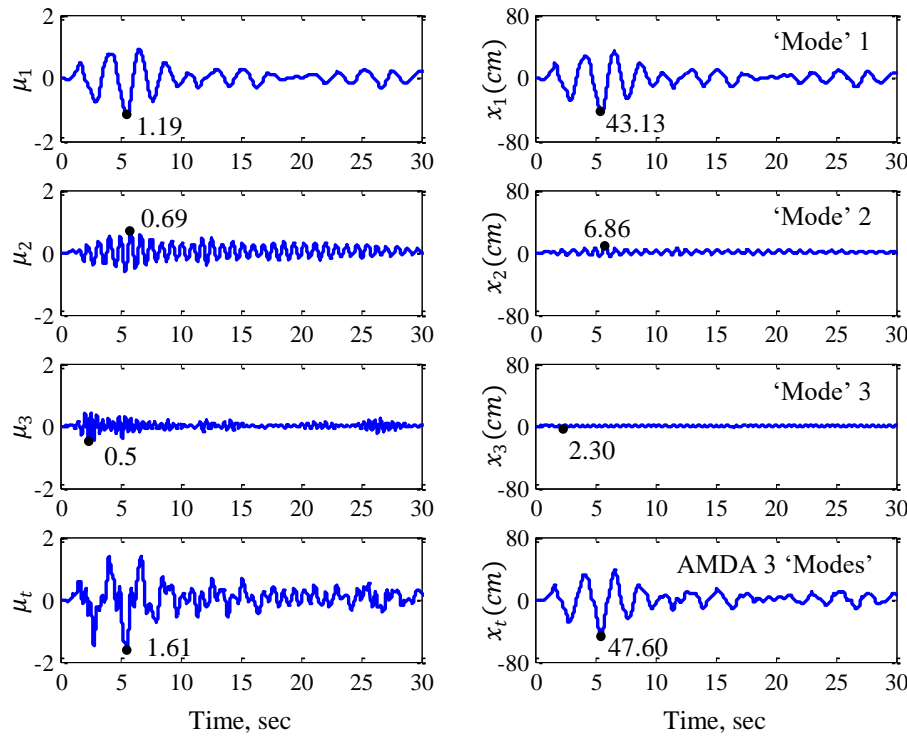


Figure. 4 Response histories of ductility demand and roof displacement from the proposed procedure for  $1.5 \times$ El Centro ground motion: first three modal responses and total (all modes) response



The approximate multimodal dynamic analysis of the structure starts with obtaining the multimodal Pushover curves of the MDOF system subjected to lateral forces distributed over the building height. In the proposed procedure, the movements will be decomposed in the form of a series of normal modes in terms of the ductility demand. Eq. (15) is solved, and the resulting ductility demand history is decomposed into its “modal” components. The obtained response histories of ductility demand and roof displacements for the three first modes of the selected building subject to 1.5 times El Centro ground motion (N/S) component ( $PGA = 0.32g$ ,  $PGV = 36.14$  cm/sec, and  $PGD = 21.34$ cm) are shown in Fig. 4.

The proposed procedure is evaluated by comparing the computed displacements histories according to Eqs. (15) - (16), considering three modes with those estimated by the NL-THA analysis and the Uncoupled Modal Response History Analysis (UMRHA) that was developed by Chopra and Goel (2001) (see Fig. 4 and 5).

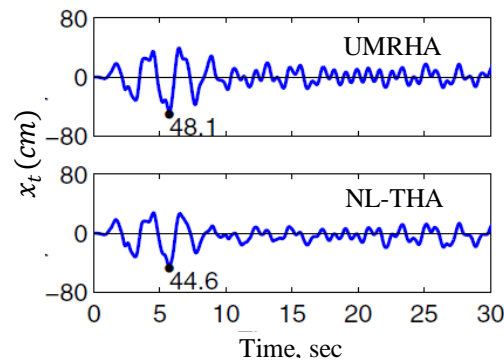


Figure. 5 Total response histories of roof displacement for  $1.5 \times$ El Centro ground motion from the UMRHA and NL-THA [7]

Following the AMDA procedure aforementioned, the total response is determined using the UMRHA and NL-THA (“exact”). Fig. 4 shows the ductility demand, also is shown in the same figure the roof displacements time histories. It is clear from the comparison shown in Fig. 4 and 5 that the AMDA gives results in good agreement with the NL-THA.

#### 4. CONCLUSIONS

An approximate procedure for seismic demands assessment of MDOF system has been developed and its accuracy was verified by examples. An inelastic modal decomposition in terms of ductility has been developed to construct the Approximate Multimodal Dynamic Analysis. That was verified using the seismic response of an example steel frame structure for which capacity curve data is available. The results indicated that more reliable displacement predictions are obtained from the proposed method.

Also, as presented in this paper, a nonlinear multimodal spectral analysis was developed. The base shear–roof displacement ( $V_{bn} - x_{rn}$ ) curve is developed from a Pushover analysis. This Pushover curve is idealized as a bilinear force–deformation relation for the  $n$ th mode of inelastic SDOF system. This idealization is used to determine the yield strength coefficient  $q_n$  and the post-to-preyield stiffness ratio  $\alpha_n$  to estimate the ductility demand.

The efficiency of the AMDA procedure is evident; the designer needs only to have the Pushover curve of the structure and the design earthquake(s) to determine peak response of any structure, namely, base displacement and base shear. This method is applicable to a variety of uses such as a rapid evaluation technique for a large inventory of



buildings, a design verification procedure for new construction, an evaluation procedure for an existing structure to identify damage states.

The results of the AMDA method are acceptable while compared to the nonlinear dynamic analysis results. For structures that the first mode contributes significantly to the response, the AMDA method will generally give good estimates of demand for global deformations. However, inelastic dynamic response may differ significantly from a selected or from the response distributions with constant lateral loads. For example, we can expect significant differences in structural response due to the influence of vibration modes at higher frequencies.

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