



### STOCHASTIC VARIABLES IN MODELLING OF THE WAVE LOADS ON OFFSHORE WIND TURBINE STRUCTURES

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**Abstract.** Nowadays, with respect to trend of Kyoto protocol, many producers turn to renewable energy resources. That leads to a fact that more than 75% of new power capacity installations in EU in the year 2015 are renewables. The leading among the new renewable energy resources is wind energy. In the last decades, even more wind energy is accommodated by moving offshore. That brings up a problem of more complicated design, which includes new loads to be investigated and modelled. For offshore wind turbines, dominant loads are wave and wind loads. For the substructure itself, the highest impacts have wave-induced loads, as it is submerged at most of its height. As the waves are stochastic and irregular loads, this paper investigates appropriate methods for modelling of the wave loads, in order to achieve a very realistic load model and results. This is only some of the numerous challenges in this area of expertise, which are more accurately investigated in a research plan within the framework of the Innovative Training Network (ITN) AEOLUS4FUTURE, related to reliability of offshore wind energy structures.

## 1 INTRODUCTION

In the last decade, the offshore wind energy industry expanded greatly in order to meet requirements of both world's growing demand for energy, and environmental-friendly solutions for energy resources. Despite of complicated and expensive transport and installation, it results with a higher electricity output compared to onshore wind turbines, due to the higher wind speeds and lower turbulence level in ocean conditions. The general trend in the offshore wind turbine industry nowadays is increase of the maximum capacity. Some of the largest offshore wind turbines today have energy output of 5MW. Recently, turbines with output of 6MW and 7MW have been installed, and in the future it is expected sizes to grow up to 20MW, if proven economically and technically feasible [6].



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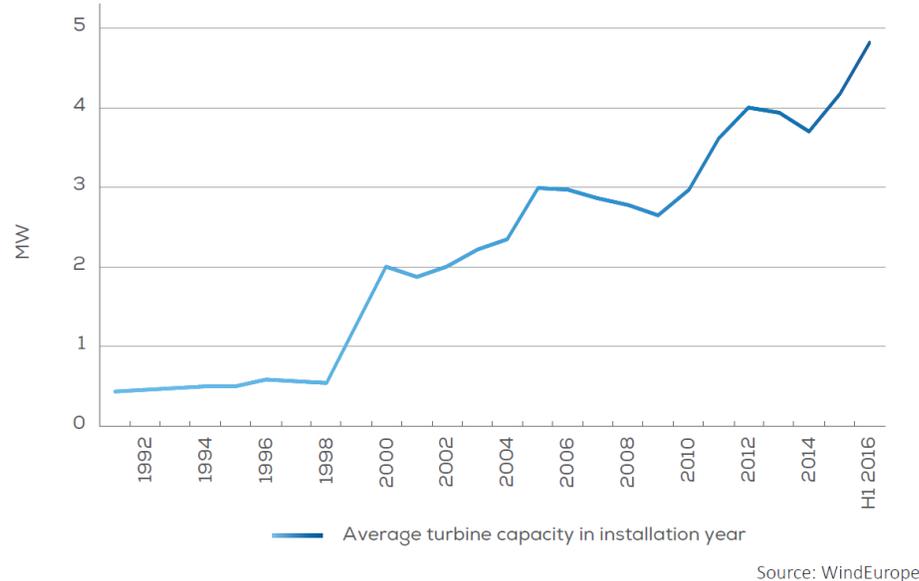


Figure 1. Increase of offshore wind turbine capacities [6]

The size increase brings up the problem of larger generators, higher hub heights and larger structures and foundations, which leads to higher dynamic complexity of offshore wind turbines. As OWTs are exposed to aerodynamic, hydrodynamic and mechanical loading, they are especially prone to fatigue damage. While the tower and blades are mostly affected by aerodynamic loads, the substructure is mostly influenced by wave-induced loads, as it is submerged at most of its height. As the OWTs are structures with significant dynamic response, the modelling of the sea surface and its kinematics is very important for the design process. The wave loads are very stochastic and contain lots of uncertainties and randomness, which are treated through different methods within the modelling process. This paper investigates pros and cons of some of the possible methods for wave load modelling, considering the vicinity of the model to reality, covering of randomness in variations of wave characteristics, design time consumption, and design driving criteria.

## 2 METHODS

There are two methods to describe and model the wave conditions that are to be considered for structural design purposes: by deterministic design wave methods or by stochastic methods applying wave spectra. For quasi-static response of structures, it is sufficient to use deterministic regular waves characterized by wavelength and corresponding wave period, wave height and crest height. Structures with significant dynamic response require stochastic modelling of the sea surface and its kinematics by time series. A sea state is specified by a wave frequency spectrum with a given significant wave height, a representative wave peak period or frequency, a mean propagation direction and a spreading function. In application, the sea state is usually assumed a random process that is stationary over a certain period of time. Three hours has been introduced as a standard period of sea state stationarity, but depending on the conditions and needs of analysis, it can range from 30 minutes to 10 hours [1,2,4].



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### 2.1. Regular waves

There are several theories in literature to describe the kinematics of regular, two-dimensional wave. Wave parameters that determine which wave theory to apply in a specific problem are the wave height  $H$ , the wave period  $T$  and the water depth  $d$ . These parameters are used to define following non-dimensional parameters that determine ranges of validity of different wave theories [2]:

- Wave steepness parameter

$$S = 2\pi \frac{H}{gT^2} \quad (1)$$

- Shallow water parameter

$$\mu = 2\pi \frac{d}{gT^2} \quad (2)$$

Figure 2 shows a diagram, taken from [1], which gives guidance on the selection of an appropriate wave theory depending on wave steepness parameter and shallow water parameter:

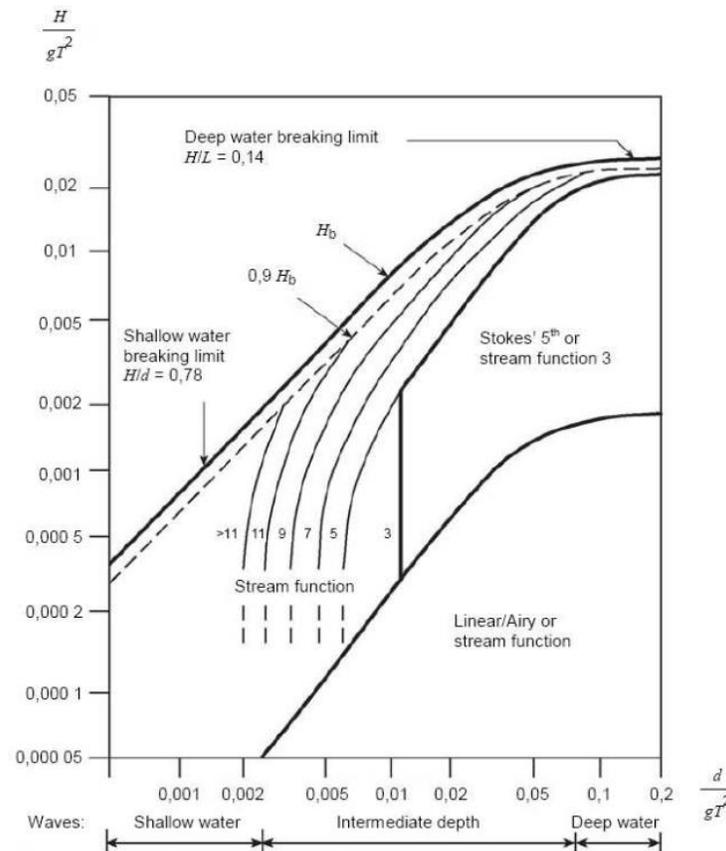


Figure 2. Regular wave theory selection diagram [1]



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### 2.1.1. Linear waves

The linear wave theory, in the literature also known as small amplitude wave theory, sinusoidal wave theory or airy wave theory, is the simplest wave theory obtained by taking the wave height to be much smaller than both the wavelength and the water depth. With the circular frequency of the wave  $\omega$ , and the wave height  $H$ , water surface elevation,  $\eta$ , can be expressed as follows [1,2,4]:

$$\eta = \frac{H}{2} \cos(\omega * t) \quad (3)$$

where:

$$\omega = \frac{2\pi}{T} \quad (4)$$

For linear waves in deep water, particle kinematics are given through the following equations:

$$v_h = \frac{H}{2} * \omega * e^{kz} \cos(\omega * t) \quad (5)$$

$$v_v = \frac{H}{2} * \omega * e^{kz} \sin(\omega * t) \quad (6)$$

$$a_h = \frac{H}{2} * \omega^2 * e^{kz} \sin(\omega * t) \quad (7)$$

$$a_v = \frac{H}{2} * \omega^2 * e^{kz} \cos(\omega * t) \quad (8)$$

where  $v_h$  stands for horizontal velocity,  $v_v$  vertical velocity,  $a_h$  horizontal acceleration,  $a_v$  vertical acceleration,  $z$  is water depth and  $k$  is the wave number, defined by:

$$k = \frac{2\pi}{L} \quad (9)$$

Wave theories for regular waves are valid only up to the still water level. For estimating the wave kinematics between the wave crest and the still water level, stretching or extrapolation methods can be used. The stream function theory (nonlinear waves) provides wave kinematics all the way up to the free surface elevation. For linear waves, common approach for estimating the wave kinematics for parts above the still water level is the method suggested by Wheeler [2,4]. The basic principle is that from a given free surface elevation record, the velocity for each frequency component is computed using linear theory, and for each time step in the time series, the vertical coordinate is stretched according to:

$$z = \frac{z_s - \eta}{1 + \frac{\eta}{d}} \quad (10)$$

where:  $\eta$  is the surface elevation,  $d$  is water depth, and  $z_s$  is the  $z$ -coordinate of the point that the wave kinematics should be stretched to. This method is graphically illustrated on the Figure 3:

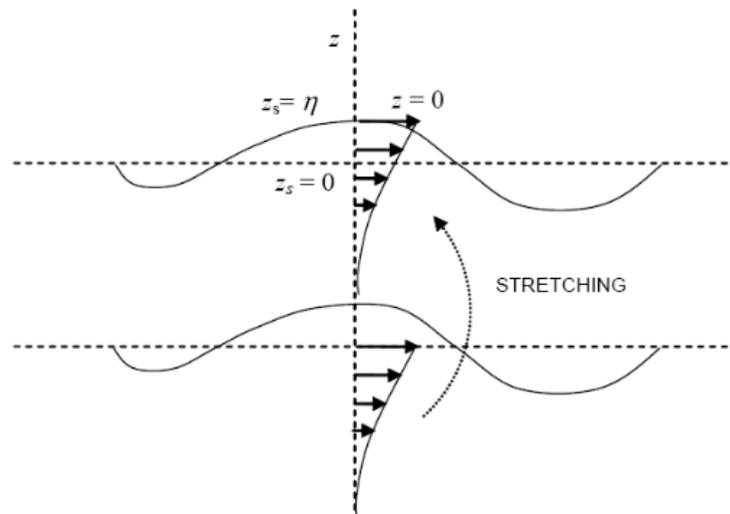


Figure 3. Stretching method by Wheeler [2]

### 2.1.2. Nonlinear waves

In case that the given wave steepness and shallow water parameters show the values out of the airy waves domain on the diagram from Figure 2., nonlinear terms must be included into the equations for calculating of the wave kinematics. This is most commonly done by applying the Stream Function Wave Theory.

As the main focus of this paper are stochastic analysis with wave load modelled as irregular sea states, no deeper explanation about nonlinear wave theories is given. For further nonlinear wave theories, a reference is made to [2].

## 2.2. Irregular sea states

Slender structures with significant dynamic response, as offshore wind turbines, require stochastic modelling of the sea surface and its kinematics by time series. The real sea states can be described as stochastic processes that are stationary over a certain period of time. Usually, the assumption is made that over a time span of three hours a sea state is stationary. The characteristics of a stationary sea state can be modelled by means of wave energy spectra. Wave spectra can be given in table form, as measured spectra, or by a parameterized analytic formula. The most appropriate wave spectrum depends on the geographical area with local bathymetry and the severity of the sea state. Models of wave spectra formulations that depend on characterizing parameters of a sea state, the significant wave height and the zero-up-crossing period, are used. These parameters are explained according to [2]:

- *Significant wave height  $H_s$*

It is defined as the mean value of the 1/3 biggest wave heights recorded in the observed time series. This value is usually shown in scatter diagrams obtained from measurements on real sites. Some investigation have shown that the significant wave height value is very close to the four times standard deviation of the surface elevation  $\sigma_\eta$ .



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- Mean zero-up-crossing period  $T_z$

It is a mean period of all successive up-crossings of the zero-water level of the water surface within the time series. The wave energy spectra usually depend on the peak period  $T_p$ , which is the period that contains the greatest amount of wave energy spectrum, and this value is most commonly given in scatter diagrams obtained from measurements on real sites. The relation of  $T_z$  and  $T_p$  depends on the shape of the spectrum and can only be established by approximate means. IEC 61400-3 [1] recommends the following correlation:

$$T_p = T_z \sqrt{\frac{11 + \gamma}{5 + \gamma}} \quad (12)$$

where  $\gamma$  is spectrum shape parameter that depends on the chosen energy spectrum. The most frequently applied spectrums for describing the wind seas are Pierson-Moskowitz (PM) spectrum and JONSWAP spectrum. The PM-spectrum was originally proposed for fully developed sea. The JONSWAP spectrum extends PM to include fetch limited seas, describing developing sea states. Both spectra describe wind sea conditions that often occur for the most severe sea states.

### 2.2.1. Pierson-Moskowitz (PM) spectrum

The Pierson-Moskowitz (PM) spectrum is used for fully developed sea states with unlimited fetch and unlimited duration of wind exposure. It is given as a function of angular frequency by:

$$S_{PM}(\omega) = \frac{5}{16} H_s^2 \omega_p^4 \omega^{-5} \exp\left(-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^{-4}\right) \quad (13)$$

where  $\omega_p$  is angular spectral peak frequency given by:

$$\omega_p = \frac{1}{T_p} \quad (14)$$

### 2.2.2. JONSWAP spectrum

The JONSWAP spectrum is formulated as a modification of the Pierson-Moskowitz spectrum for a developing sea state in a fetch limited situation. It is based on the PM-spectrum, extended by the shape parameter  $\gamma$ :

$$S_{JS}(\omega) = nf S_{PM}(\omega) \gamma^{\exp\left(-\frac{1}{2} \left(\frac{\omega - \omega_p}{\sigma \omega_p}\right)^2\right)} \quad (15)$$

where:

nf stands for normalizing function given by:

$$nf = 1 - 0.287 \ln(\gamma)$$



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$\sigma$  is bandwidth parameter, given by:

$$\begin{aligned}\sigma &= \sigma_a \text{ for } \omega \leq \omega_p \\ \sigma &= \sigma_b \text{ for } \omega > \omega_p\end{aligned}$$

$\gamma$  is shape parameter. For  $\gamma = 1$ , the JONSWAP spectrum is equal to PM-spectrum. Average values for the JONSWAP experiment data are  $\gamma = 3.3$ ,  $\sigma_a = 0.07$ ,  $\sigma_b = 0.09$ . The JONSWAP spectrum is expected to be a reasonable model for:

$$3.6 < \frac{T_p}{\sqrt{H_s}} < 5 \quad (16)$$

where  $T_p$  is in seconds and  $H_s$  in meters, and should be used with caution outside this interval [1,2,4].

### 2.2.3. Numerical simulation of irregular sea states

There is a number of models proposed to simulate irregular sea states based on wave energy spectra numerically. Theoretically, this can be done by an inverse Fourier transformation of the spectrum from the frequency domain back to the time domain. In practice, irregular random waves, representing a real sea state, can be modelled as a summation of many partial wavelets with different amplitudes, angular frequencies and phase angles. The simplest random wave model is the linear long crested wave model given by:

$$\eta(t) = \sum_{i=1}^N a_i * \cos(\omega_i t + \varphi_i) \quad (17)$$

where  $a_i$  are the amplitudes of the partial waves,  $\omega_i$  are the angular frequencies of the partial waves and  $\varphi_i$  are randomly distributed phase angles within the interval  $[0, 2\pi]$ . The same superposition model is valid for superposition of the linear partial waves' kinematics (velocities and accelerations) [2,4].

In order to obtain the angular frequencies  $\omega_i$  of the partial waves, the wave energy spectrum needs to be discretized into slices  $\Delta\omega_i$ . Several methods exist to carry out this discretization: dividing the spectrum into slices with constant width  $\Delta\omega$  (very high numerical costs for longer random time series), irrationally distributed frequency intervals (time series are no longer periodic), or frequency interval chosen such that all partial waves have the same amplitude  $a_i$  [4]. Satisfying solution is obtained by combining the last two methods for different regions of wave energy spectrum, proposed by Kleineidam (2005).

### 2.3. Wave load

Once the wave particle kinematics are determined, it is possible to calculate wave-induced loads. The most widely used empirical formula in offshore wind industry for that purposes is Morison's equation. The requirement for applicability of Morison's equation is the hydrodynamic transparency of the structure, i.e. the structure influences the wave flow only locally, without obstructing the free flow on a global scale. It is assumed that this is the case if the following condition is fulfilled:



$$D \leq \frac{\lambda}{5},$$

where  $D$  is the structure's diameter, and  $\lambda$  is wave length. For structures that do not fulfil this condition, other methods of wave load calculation have to be taken into account to consider the diffraction of the flow [1]. Otherwise, Morison's equation, as following, is used:

$$f = f_d + f_m = 0.5 * C_d * \rho_{water} * D * u_{\perp} * |u_{\perp}| + C_m * \rho_{water} * A * \dot{u}_{\perp} \quad (18)$$

where:  $f$  is force per unit length of the member,  $f_d$  is drag term of the wave force,  $f_m$  is inertia term of the wave force,  $C_d$  is hydrodynamic drag coefficient,  $C_m$  is hydrodynamic inertia coefficient,  $\rho_{water}$  is water density,  $D$  is diameter of the member in the respected section,  $A$  is cross section area of the member,  $u_{\perp}$  is velocity of the flow normal to the member surface and  $\dot{u}_{\perp}$  is acceleration of the flow normal to the member surface [1,2,4].

The values for the hydrodynamic coefficients  $C_d$  and  $C_m$  can only be determined by experiments, but approximate values can also be extracted from the literature or model tests. They depend on the flow conditions around the structure, the structural shape and the roughness of the structure. The main parameters governing the coefficients for cylindrical members are Reynolds number, relative roughness and Keulegan-Carpenter number. Recommended values for hydrodynamic coefficients depending on the Reynolds number and relative roughness, according to [3] are:

Reynolds number	Smooth cylinder		Rough cylinder	
	$C_d$	$C_m$	$C_d$	$C_m$
$\leq 2 * 10^5$	1.2	2.0	1.2	2.0
$> 2 * 10^5$	0.7	2.0	1.1	1.6

Table 1 : Indicative hydrodynamic coefficients for cylindrical members [3]

where Reynolds number is defined as:

$$Re = u_{D,max} * \frac{D}{\nu} \quad (19)$$

with  $u_{D,max}$  as maximal velocity normal to the member and  $\nu$  kinematic viscosity of the water.

### 3 RESULTS

Numerical simulations of both regular waves and irregular sea state are conducted in order to compare the stress results. A monopile OWT structure is modelled in FE analysis tool Poseidon, developed at the Institute for Steel Construction of Leibniz Universitaet Hannover. Site characteristics significant wave height  $H_s$  and wave peak period  $T_p$  are defined as 4m and 8s, respectively, which are also the characteristics of the wave energy spectra used for modelling of sea state [5]. The regular wave was designed according to [1] with respect to the following:



$$H_{50} = 1.86 * H_{s50} \quad (20)$$

$$11.1\sqrt{H_{s50}/g} \leq T \leq 14.3\sqrt{H_{s50}/g} \quad (21)$$

where  $H_{50}$  is extreme wave height with recurrence period of 50 years, recommended by [EC] for design of the extreme deterministic wave,  $H_{s50}$  is significant wave height for 4 hours reference period, and T is corresponding wave period to be taken into combination with extreme wave height  $H_{50}$ .

Both regular waves and irregular sea state were simulated for a duration of 150 s with time step of 0.2 s. Stress sensors were positioned on the embedment point of the monopile structure, at the mudline level. Stresses were measured in the direction of wave progress. Recordings of stress sensors for both regular wave and sea state simulation are shown in Figure 4:

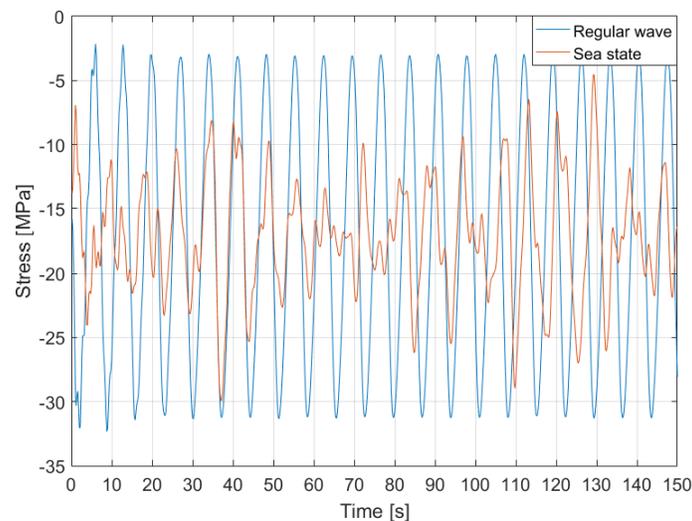


Figure 4. Stress results for regular wave and sea state model simulation in time domain

#### 4 CONCLUSIONS

By comparing the stresses obtained by simulations of regular waves with the ones induced by sea state model, it can be seen that the stress amplitudes are higher for regular waves. The reason for that is, that the regular wave is designed following the instructions from [EC] for extreme design wave. In order to retain safety level in design, due to the significant simplifications in wave load modelling, extreme wave height with period of recurrence of 50 years is used. That gives higher, but constant amplitudes of stress in time. Irregular sea state is designed as stochastic process in order to develop a model closer to real conditions, while the design driving criteria is mostly fatigue, not the extreme loads. Therefore, the stress amplitudes are mostly lower compared to extreme design wave, although for longer time simulations, where the randomness of the sea states can be better expressed, stress amplitudes of sea states can also go close to the extreme limit, as the irregular sea states are designed to consider the severity of the sea.



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It is a matter of design case, requirements and conditions, which load model shall be used. For significant research in this area, it is recommendable to design both extreme and irregular wave loads, in order to verify the resistance of structure to all limit states. However, since the OWTs are dynamically complex systems exposed to aerodynamic and hydrodynamic cycle loading, they are especially prone to fatigue damage. Therefore, at most cases it is recommendable to model the wave load by means of sea states.

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