DAMPING MODELS IN CONTINUUM METHODS FOR DYNAMIC ANALYSIS OF BUILDINGS

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Keywords: *passive damping, replacement beams, structural dynamics*

ABSTRACT

Condensed 1D continuum models, called "Replacement Beams" (RBs), often provide an useful tool to capture basic features of the static response of a wide class of regular buildings, among which high rises. Despite potential capabilities of RB models, up to now few attempts have been done toward their adoption in dynamical calculations considering structural damping, as well as in modelling passive-damping devices, more and more frequently provided in order to attain comfortable, robust and resilient structural arrangement in hazard prone conditions with sustainable constructional indexes and material savings. The introduction of simple damping mechanisms and an overview on potentialities in passive damping optimization analysis are therefore briefly outlined in order to show potentialities of these approaches in producing feasible dynamic estimations.

1 INTRODUCTION

While requirements for structural safety and comfort under uncertain external loading and environmental conditions are continuously growing in first world countries, "*structural systems need to avoid excessive material use to counteract the forces of nature, and methods which are harmonious and even synergetic with forces of nature need to be developed and incorporated for sustainable design*" [1]. This need is clearly emphasized where the structure itself represents the main component of the construction, as it happens in mid- and high-rise buildings where the so called "premium for height" refers to building's sensitivity to demanding wind and earthquake loadings.

As dynamics becomes the main driver in structural dimensioning, current design trends are aimed at reducing its effects by increasing dissipation through the adoption of supplemental damping devices, following the so called "passive damping approach to structural control" [2]. These damping devices are Viscous or Friction Dampers (VDs, FDs), introduced across points with appreciable relative motions, or Inertial Dampers, like Tuned Mass Dampers (TMDs) and Tuned Liquid Dampers (TLDs). Hybrid damping is a more recent and promising approach for multi-hazard structural control, implying use of VD's, supplemented by a TMD generally placed at the top of the structure [3,4,5]. Adoption of these structural enhancement strategies reflects in a reduction of structural materials, while increasing safety, robustness and resilience, so that passive damping gets a role from the viewpoint of structural sustainability [6]. A lot of research has been dedicated in recent years to effectiveness and optimality assessment in passive damping strategies, in particular to the problem of optimal placing of dampers. Despite robust numerical approaches have been proposed and successfully adopted [e.g. 7,8], applications to complex building arrangements reveal quite cumbersome for current practice, frequently jeopardising the intuition-based approach that should drive the common engineering sense, especially during conceptual and preliminary design phases. Simplified approaches to building dynamics could therefore provide a valuable tool to emphasize basic features of the structural response, pre- and post- passive damping application.

It is well known that continuum methods like Replacement Beam (RB) approaches [9-13], provide an useful and feasible approximation tool to analyse buildings at conceptual level, drastically simplifying the complexities of the analyses at that stage and leaving 3D FEM modelling to subsequent design phases. Depending on the structural characteristics of the building, several different RB theories can be referred to, characterized by proper kinematical models and equivalent stiffness parameters, which represent the real stiffness of the system as a whole. RBs usually provide static response at global level and are also a feasible tool to define fundamental dynamic eigenproperties. It's however worth noticing that up to date the research works carried out on RBs seldom deal with the full dynamic response (i.e. time-histories) including dissipation mechanisms. A consistent definition of damping models for the different RB's is however of fundamental importance, not only for a proper analysis of the base structure, but also in order to deal with passive damping devices framed within the same continuous approach. This communication therefore summarizes some results of recent research efforts aimed at generalise the class of RBs approaches by including consistent definitions of relevant damping mechanism, suitable to capture both the response of the base structure and the effects of introduction of passive damping within formulations, enabling simple optimisation algorithms for optimal device distributions. For a necessary sake of shortness, hybrid damping strategies are not herein discussed.

2 CONTINUUM MODELS FOR BUILDING ANALYSES

RBs formulations are often able to capture fundamental features of the horizontal dynamic response of a wide class of regular buildings acted upon by seismic or wind actions. The first step of this idealisation approach requires the definition of a relevant RB model, dependent upon the specific structural arrangement (Fig.1).

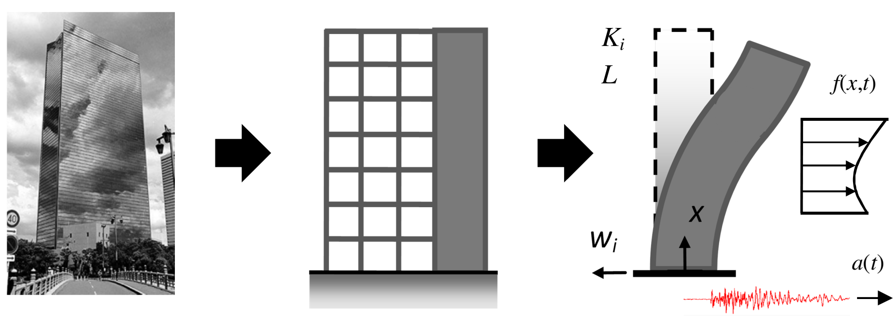

Fig. 1 – Synthesis of the RB-idealisation process

Classical RBs are (Fig. 2): (a) *Bernoulli's beam* (BB), suitable for a first approach in modelling tall buildings; (b) *Shear beam* (SB), able to capture behaviour of shear-type buildings; (c) *Coupled Two Beams* (CTB), a parallel coupling of BB and SB, reveals appropriate for coupled shear-walls and shear wall-frame structures; (d) *Timoshenko's beam* (TB), a series coupling of BB and SB, properly models shear walls or trussed resisting schemes; (e) *Sandwich Beam* (SWB), equivalent to the parallel coupling of TB and BB, is the most general framework, able to describe the fundamental behaviour of frames coupled to single or multiple shear-walls. The large applicability of SWB to all the conventional structural schemes depends on its capability to describe the three fundamental kinematical modes of a general building system (Fig. 3).

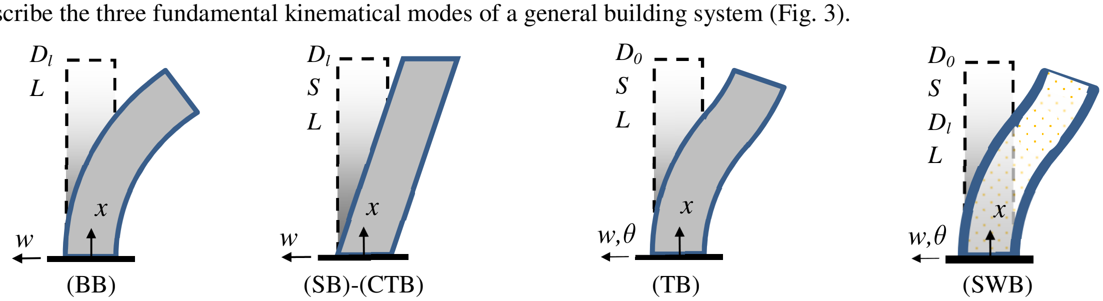

Fig. 2 - Classical RB models

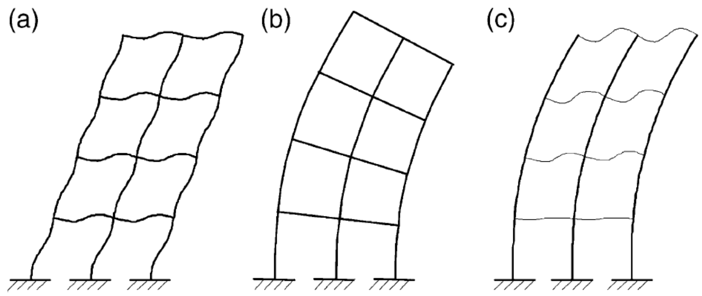

Fig. 3 – Modes of deformations of a general building (from [10]):
(a) shear; (b) global bending; (c) local bending

In all the cases the equations of (transverse) motion are obtained throughout application of Hamilton's Principle to the relevant Lagrangean function $\mathcal{L}(q,\dot{q})$ (where $q,\dot{q}$ are the generalised displacement and velocity fields for the specific model):

$$\delta\int_{t_0}^{t_1}\mathcal{L}(q,\dot{q})\,dt=\delta\int_{t_0}^{t_1}\left(\mathcal{T}-\mathcal{V}+\mathcal{W}\right)dt=0 \tag{1}$$

$\mathcal{T},\mathcal{V},\mathcal{W}$ are the kinetic energy, the strain energy and the work exerted by external loading system, respectively. The following definitions for strain energies hold in particular for CTB, TB and SWB:

$$\mathcal{V}_{CTB}(v)=\frac{1}{2}\int_0^L\left(D_l v''^2+S\,v'^2\right)dx \tag{2}$$

$$\mathcal{V}_{TB}(\vartheta,v)=\frac{1}{2}\int_0^L\left[D_0\vartheta'^2+S(\vartheta-v')^2\right]dx \tag{3}$$

$$\mathcal{V}_{SWB}(\vartheta,v)=\frac{1}{2}\int_0^H\left[D_0\vartheta'^2+D_l v''^2+S(\vartheta-v')^2\right]dx \tag{4}$$

with (v,ϑ) describing the transversal displacement and the local rotation of the reduced 1D formulation.
When rotatory inertia effects are neglected, the same expressions apply for all the three formulations, i.e.

$$\mathcal{T}=\frac{1}{2}\int_0^L\rho\,\dot{v}^2\,dx,\mathcal{W}=\int_0^L\left[f(x,t)-\rho(x)a_g(t)\right]v(x)\,dx \tag{5,6}$$

Bending (D_l, D_0) and shear (S) parameters are introduced for every specific structural arrangement, for example through the relations suggested in [9]. These approaches provide an useful tool for a feasible evaluation of static deflections and/or fundamental frequencies. Due to the inherent decomposition into elementary modes of deformations, the same global static parameters and frequencies can be also respectively determined through series/parallel stiffness compositions rules and the so-called Summation Theorems (in this case, Southwell and Föppl formulas [14]), applied to the previously stated kinematic decompositions:

$$\text{CTB} = \text{BB} \parallel \text{SB};\ \text{CTB} = \text{BB} \times \text{SB};\ \text{SWB} = \text{TB} \parallel \text{BB} = (\text{BB} \times \text{SB}) \parallel \text{SB} \tag{7}$$

where formal operators $\parallel$ and $\times$ means parallel and series coupling, respectively.

3 DAMPING IN CONTINUUM MODELS

As previously recalled, with very few exceptions, researches carried out on RBs during the last two decades were almost exclusively focused on the estimation of (engineering) modal quantities of building structures and none proposal has been carried out on dynamical models embedding dissipation effects. However, the promising adoption of a full-mechanism RB models as a tool for dynamical analyses of regular buildings (not necessary tall), some investigations have been recently oriented toward the extension of these formulation aimed at including kinematically coherent material and structural damping mechanisms [15-18].

Although it is nowadays well assessed that structural damping has an inherently friction-type, frequency-independent and amplitude-dependent nature [19-24], equivalent viscous models still provide a direct and simple (linear) tool to describe and understand basic features of single structural dissipation mechanisms, as to model additional damping devices, especially when optimisation procedures have to be dealt with [7,8]. If this kind of dissipation model is referred to, a proper Rayleigh function $\mathcal{R}(\dot{q})$ can be defined and the governing systems is associated to the stationary condition for the modified Hamiltonian

$$\delta\int_{t_0}^{t_1}(\mathcal{L}-\mathcal{R})\,dt=\delta\int_{t_0}^{t_1}(\mathcal{T}-\mathcal{V}+\mathcal{W}-\mathcal{R})\,dt=0 \tag{8}$$

The $\mathcal{R}-$function can be introduced for all the previously listed classical models when referring to a linear visco-elastic behaviour at material level, i.e.

$$\begin{cases}\sigma=\sigma^e+\sigma^d=E\varepsilon+c_b\dot{\varepsilon}\\ \tau=\tau^e+\tau^d=G\gamma+c_s\dot{\gamma}\end{cases} \tag{9}$$

where symbols are self-commenting, and

$$\oint(\sigma\dot{\varepsilon}+\tau\dot{\gamma})=\oint(\sigma^d\dot{\varepsilon}+\tau^d\dot{\gamma})=\mathcal{D}(\dot{\varepsilon},\dot{\gamma})=2\mathcal{R}(\dot{\varepsilon},\dot{\gamma})>0 \tag{10}$$

In particular, features of bending and shear damping, depending upon curvature and shear strain velocities,

$$\mathcal{R}_b=\tfrac{1}{2}\int_0^L c_b I\,\dot{\chi}^2dz;\mathcal{R}_s=\tfrac{1}{2}\int_0^L \kappa c_s A\,\dot{\gamma}^2dz \tag{11a,b}$$

have been investigated for both TBs [16-18] and CTBs/SWBs [25], considering their physical significance as depending upon the specifically considered structural model. When required, these contributions is superimposed to the classical viscous contribution

$$\mathcal{R}_v=\tfrac{1}{2}\int_0^L C\,\dot{v}^2dz \tag{12}$$

A quite intuitive proof given in [18,25] states that for low "mechanism-dependent" damping ratios (i.e. < 5% for shear, bending or classical), all these models produce the same structural effects, being their distinctive features appreciable only where higher ratios introduce modification of (non-classical) eigen-modes with respect to those of the corresponding udamped response. This problem has been specifically studied for TBs (Fig. 4-5) and for CTBs and SWBs regarded as a reference model for coupled, axially extensible, shear walls (Fig. 5-6).

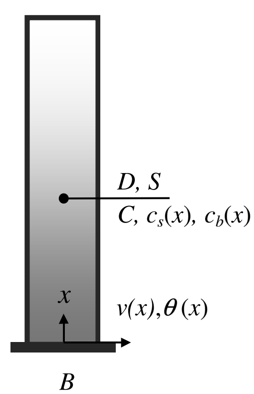

Fig. 4 – Damped TB models

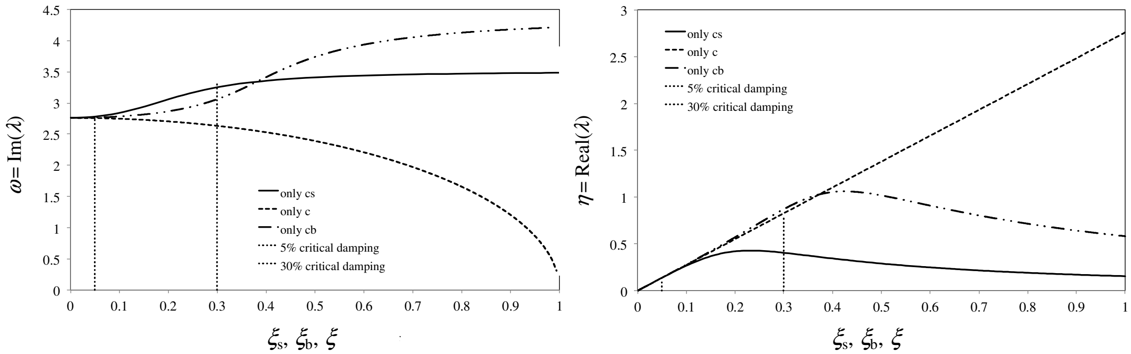

Fig. 5 - Damping mechanisms in TBs: (a) frequency and (b) damping vs damping ratios

Introducing the concept of the “equivalent continuum core” [26,27], a smeared replacement of the discrete distribution of the connecting beams (*cb*), a generalised sandwich model (GSWB) for coupled shear walls (*sw*) based over the kinematic and geometry-parameters described in Fig. 6 can be cast in a format that is substantially equivalent to that in Eq.(4), but depending now upon three kinematic fields provides a more meaningful formulation. When an homogenous material is assumed, Potential energy reads

$$\mathcal{V}_{GSWB}\left(v,\vartheta,w\right)=\frac{1}{2}\int_0^L\left[D\vartheta'^2+S(\vartheta-v')^2+Kw'^2\right]dx+\frac{1}{2}\int_0^L(k_{eq}/\ell_b^2)\left(B\theta+\ell_b v'-2w\right)^2 dx \qquad (13)$$

where $D=2EI_{sw}, S=2\kappa GA_{sw}, K=2EA_{sw}, k_{eq}=\kappa G_{eq}t\ell_b$, $G_{eq}=\frac{\ell_b}{ht}\left(\frac{\ell_b^3}{12EI_{cb}}+\frac{\ell_b}{\kappa GA_{cb}}\right)^{-1}$ and κ =5/6.

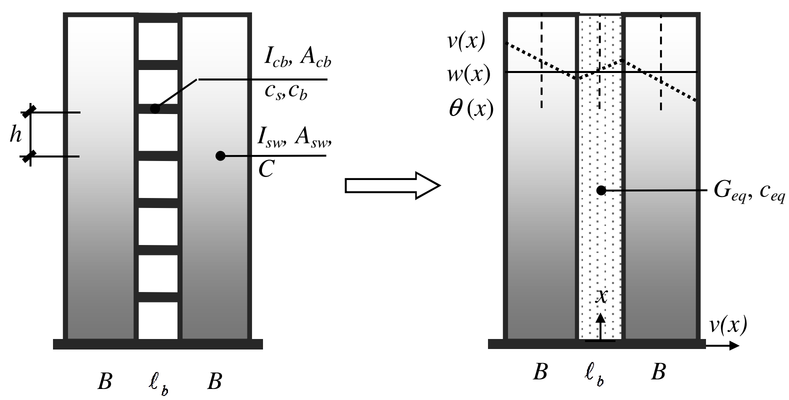

Fig. 6 - Damping effect in coupled shear walls modelled as SWB

Dissipation can be analogously inserted into this formulation by assuming a visco-elastic law for both shear walls and connecting beams. Supposing, for a sake of simplicity, to consider shear and bending damping into connecting beams only, and classical damping in shear walls, the relevant Rayleigh’s function is

$$\mathcal{R}\left(\dot{v},\dot{\vartheta},\dot{w}\right)=\frac{1}{2}\int_0^L\left[C\dot{v}^2+(\kappa c_{eq}t/\ell_b)\left(B\theta+\ell_b v'-2w\right)^2\right]dx \qquad (14)$$

The equivalent core damping c_{eq} is derived by equating the dissipation energy due to bending and shear damping c_b, c_s in a single connecting beam with the smeared dissipation into the equivalent core. Its expression reads

$$c_{eq}=\frac{\ell_b}{ht}\left(\frac{\ell_b^3}{12c_bI_{cb}}+\frac{\ell_b}{\kappa c_sA_{cb}}\right)^{-1} \qquad (15)$$

4 PASSIVE DAMPING DESIGN STRATEGIES

The effects of (partially) distributed internal viscous damping (DIVD) have been also studied for the latter models, as this dissipative configuration is of particular relevance when some piece-wise additional damping have to be considered. These continuous models allow to deal with the presence of local damping devices in a smeared and kinematically coherent way, leading to a compact formulation which allows simple approaches to the classical optimisation problems. As a matter of example the two cases of TB and SWB with DIVDs are hereafter briefly outlined with respect to the optimal placing of the additional damping devices.

In the TB case of Fig. 7, damping addition ΔC_i provided by discrete VDs are treated as an additional smeared shear damping $\Delta c_s(x)$. An optimisation problem is therefore easily formulated with respect to any particular objective function (e.g. the tip displacement) and specific excitation. Results obtained for piecewise constant distribution, $\Delta c_s(x) = \Delta \overline{c}_s$ and an harmonic transversal load (Fig. 8), highlight an optimal distribution length L_d, spreading from the clamped edge and depending upon the amount of extra damping [18].

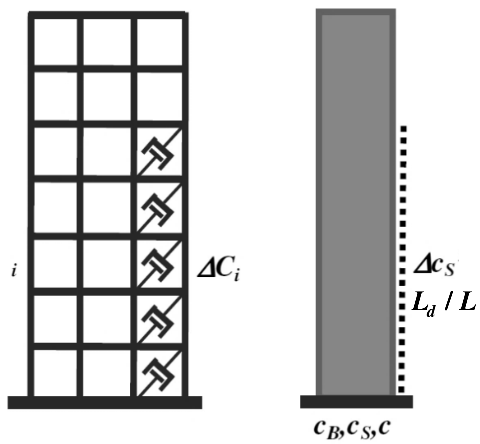

Fig. 7 – Smeared equivalent additional damping in structure modelled as TB

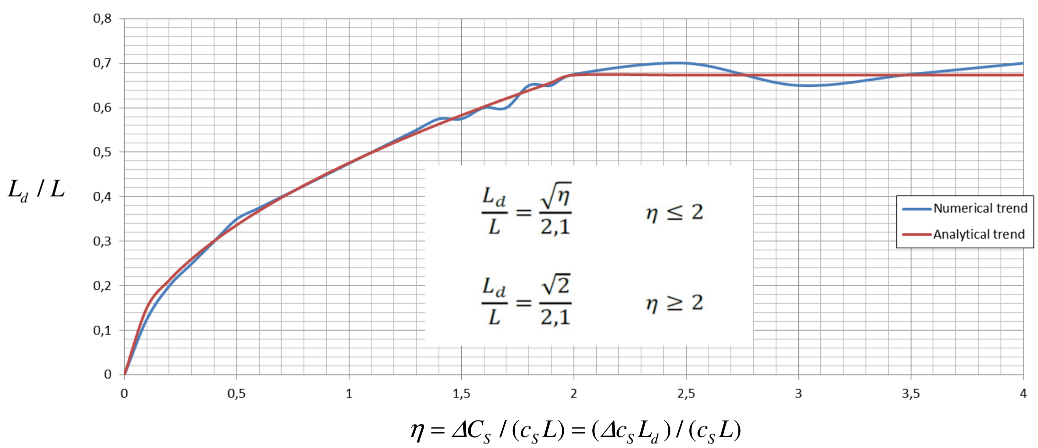

Fig. 8 – Optimal additional damping length depending on extra-damping factor η

The coupled shear walls modelled as a SWB with DIVDs, is now shortly described in Fig.9. In this case an augmented version of function Eq.(14) is introduced and an optimisation strategy is followed for the optimal placing of additional (smeared) core damping, depending upon its relative amount with respect to the base one in both shear walls and connecting beams. Unlike in TBs, core damping in SWBs is maximised where the highest relative rotations between walls is experienced. For common levels of coupling, this position in placed at an height position $0.5 < x/L < 0.8$ [28], reaching the tip only at the uncoupled or fully coupled limits.
Optimisation results (Fig. 10) give therefore the optimal distribution length L_d and the equivalent damping ratio (where a classical damping C corresponding to 5% is adopted for the reference structure) as a function of extra damping ratio η and a coupling parameter ε [28]. Is worth noticing that in this case the definition of η is different from the one defined in TB case:

$$\eta = [(c_{eq} t \ell_b) L_d] / C(5\%) \tag{16}$$

Results reveal that the optimal damping configuration is strictly confined around the maximum rotation point up to 15% damping ratio, while for higher values the optimal distribution length spreads up to 0.5L.

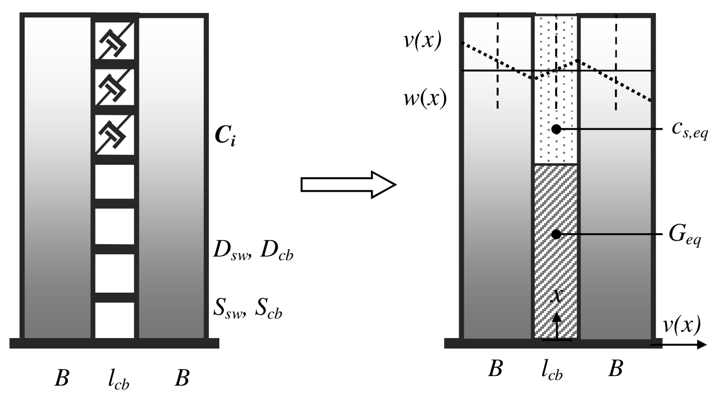

Fig. 9 – Additional damping effect in coupled shear walls modelled as SWB

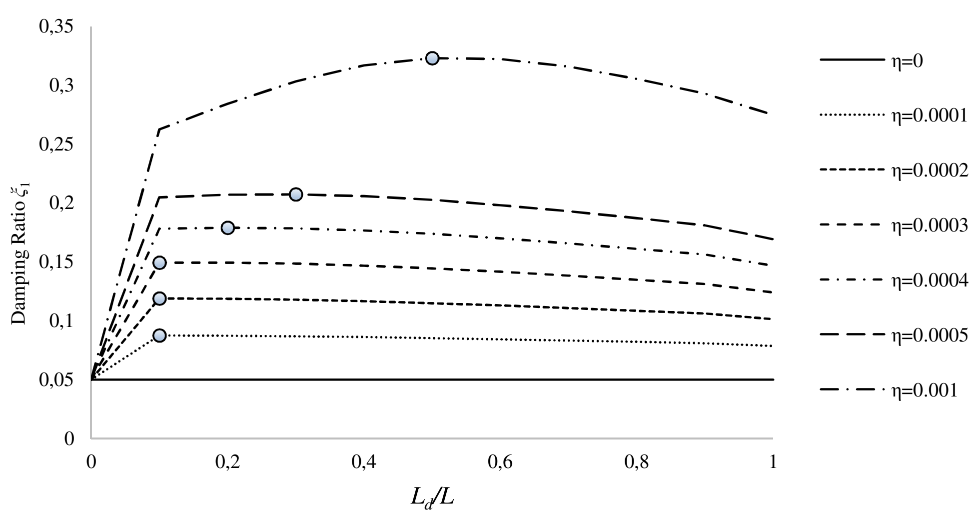

Fig. 10 – Optimal additional damping length placed around the maximum rotation point depending on extra-damping factor η

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