Civil Engineering for Sustainability and Resilience International Conference, **CESARE '14**
Amman, Jordan, 24-27 April 2014

INVERSE TECHNIQUE FOR INVESTIGATION OF THE POST-CRACKING BEHAVIOUR OF SFRC MEMBERS IN FLEXURE

G. Kaklauskas[1], V. Gribniak[1], A. Meskenas[1], V. Gelazius[1] and A. Rimkus[1]

[1]Department of Bridges and Special Structures, Vilnius Gediminas Technical University,
Sauletekio av. 11, LT-10223 Vilnius, Lithuania
e-mail: gintaris.kaklauskas@vgtu.lt

Keywords: *Residual stresses, Steel fibre reinforced concrete, Flexural members, Crack width.*

ABSTRACT

Steel fibre reinforced concrete (SFRC) is a cement-based material reinforced with randomly distributed fibres. The inclusion of fibres into the concrete matrix counteracts its brittleness, producing material with increased ductility, toughness and post-cracking stiffness as well. Due to the ability of fibres to transfer stresses through the crack plane, cracked SFRC is able to carry a certain portion of tensile stresses. Therefore, post-cracking strength can be considered as the main parameter describing the effectiveness of fibres and concrete interaction. Stresses resisted by cracked SFRC are known as residual and can vary significantly depending on the fibre amount and mechanical properties of fibres and concrete.

The paper deals with experimental and theoretical investigation of the post-cracking behaviour of SFRC. Test results of six notched beams with fibre contents of 0.5 and 1.0% by volume subjected to three-point loading are presented. Considering SFRC as a homogeneous material, the inverse analysis technique is proposed for determination of the residual stresses of SFRC in tension. To verify the constitutive modelling results, a numerical simulation was utilized employing a nonlinear finite element analysis program ATENA. Simulated load-crack width curves were compared with the test data validating adequacy of the proposed technique.

1 INTRODUCTION

Steel fibre reinforced concrete (SFRC) has become a widespread material in areas such as underground shotcrete structures and industrial floors. However, due to the absence of universally accepted material models, reliable for numerical analysis, application fields of SFRC are still limited.

The inclusion of fibres into the concrete matrix contributes mainly to the energy absorption capacity and crack control leading to the increased ductility, toughness and post-cracking stiffness of the structural elements [1, 2]. Steel fibre reinforcement becomes chiefly effective after the concrete cracking initiation and, mostly, improves the post-cracking behaviour, due to the stress transfer mechanisms provided by fibres bridging cracked sections. Crack propagation in SFRC is counteracted by the bond stresses that develop at the fibres and concrete matrix interface during the fibre pull-out. Therefore, a cracked section is able to carry tensile stresses. This effect is known as the residual stresses σ_{fr} of SFRC in tension.

One of the most important properties of SFRC is its ability to uniformly transfer residual stresses across a cracked section. However, it is strongly dependent on effectiveness of the steel fibre reinforcement, i.e. fibre properties (geometry, strength, bond with concrete, etc.), as well as fibre orientation and distribution in the cracked section [3]. The stresses σ_{fr} are often defined using results of standard bending tests [4, 5, 6, 7] often accompanied by a large scatter. Therefore, quantifying the residual stresses is the most intricate task in the constitutive modelling of SFRC [8].

SFRC may be considered as a homogenous material, thus the ability to resist tension stresses over the cracked section can be described by residual stress-crack opening (σ_{fr}-w) relationship. Residual stresses can be determined using different empirical techniques or experimental methods (uniaxial tension, wedge splitting, bending tests). Due to the difficulties in performing the uniaxial tension and wedge splitting tests [9], flexural tests on notched beams are widely used for indirect determination of residual stresses in tension of a SFRC member. Performing bending test under deformation control, the crack width and the deflection are measured together with the corresponding load applied. Attained response of the notched beam is used for the determination of σ_{fr}-w relationship through the proposed procedure of the inverse analysis.

The present investigation is dedicated to the determination of the residual stress-crack opening (σ_{fr}-w) relation of SFRC. A simple and universal method for the assessment of the residual stresses is proposed. The residual stresses were obtained using the inverse technique on the basis of the experimental data from three-point bending tests on notched members. Unlike the existing theoretical methods [1, 10], the developed technique is able to obtain the residual stresses at any loading level. Moreover, it is not limited by the standard bending specimens.

2 CONSTITUTIVE ANALYSIS TECHNIQUES

2.1 The inverse analysis

Cracked SFRC beam under three-point bending is considered as a mechanism of two rigid parts interacting at a fictitious hinge (Fig. 1). For the actual design, following assumptions were made: location of the hinge coincides with the neutral axis; elastic part of deformations is negligibly small; and crack surfaces remain plane (i.e. the overall angular deformation equals the crack-opening angle). Based on the assumptions, the crack-opening angle φ, deflection δ and the distance between crack tip and neutral axis y can be geometrically interrelated as follows:

$$tg\ \varphi = 2 \cdot \delta / L, \tag{1}$$

$$tg\ \varphi = w / 2y. \tag{2}$$

From the above equations, location of the neutral axis can be determined:

$$\frac{2 \cdot \delta}{L} = \frac{w}{2 \cdot y} \quad \Rightarrow y = \frac{w \cdot L}{4 \cdot \delta}. \tag{3}$$

The inverse technique assumes that division of entire crack into relatively small crack ranges w_i leads to constant residual stresses σ_{fr} (in a single crack range). After cracking, the undamaged tensile concrete zone is very small and does not have any influence to the post-cracking behaviour of the element, thus it is presumed that residual stresses act up to the neutral axis (Fig. 1). Determination of the residual stresses is described below.

Small constant crack intervals Δw are chosen, e.g. $\Delta w = 0.05$ mm, which in current simulation serve as the analysis steps. The experimentally determined load-crack opening diagram is divided into parts, according to the specified crack ranges w_i ($\boldsymbol{w} = \{0.05; 0.10; 0.15; \ldots, w_n\}$) and the corresponding loads P_i as well as deflections δ_i are defined (Fig.2).

Considering the *i-th* analysis (crack opening) step, the equilibrium equation of bending moments is solved with respect to the point O (Fig. 3), thus residual stress $\sigma_{fr,i}$ can be assessed:

$$\sigma_{fr,i} = \frac{M_{ext,i} - \sum_{k=0}^{i-1} \sigma_{fr,k} \cdot z_k \cdot H_i}{z_i \cdot H_i}. \tag{4}$$

Here $M_{ext,i}$ is the unit bending moment, H_i is a height of layers with constant residual stress $\sigma_{fr,i}$ $(H_i = y_i / i)$; z_i is the distance between resultant $F_{fr,i}$ and the point O.

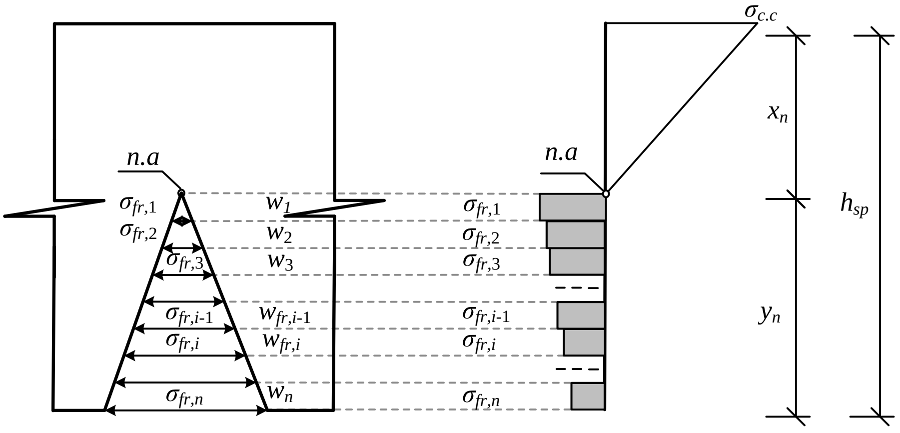


Figure 1. Assumed residual stresses distribution

The distance z_i can be found as:

$$z_i = \frac{y_i + 2\cdot y_i\cdot(i-1)}{2\cdot i} + \frac{2}{3}\cdot x_i\,, \tag{5}$$

where x_i is the height of compressive zone $(x_i = h_{sp} - y_i)$; y_i is the distance between the neutral axis and the notch tip.

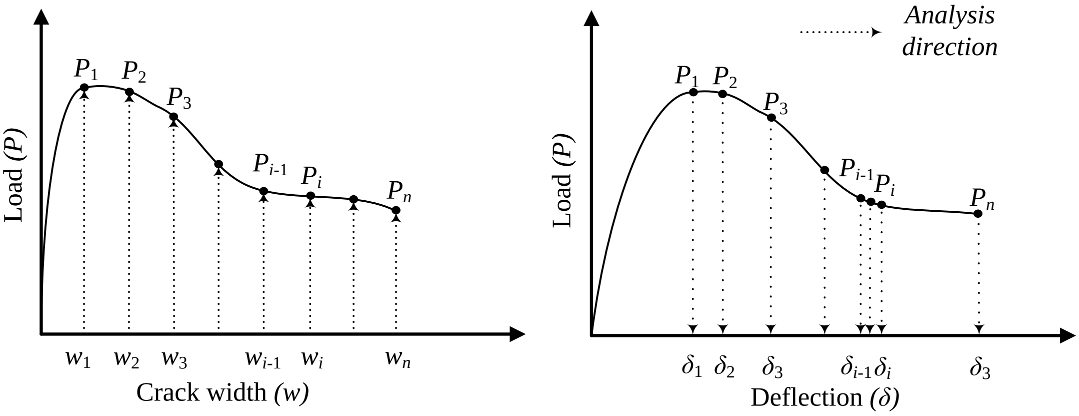
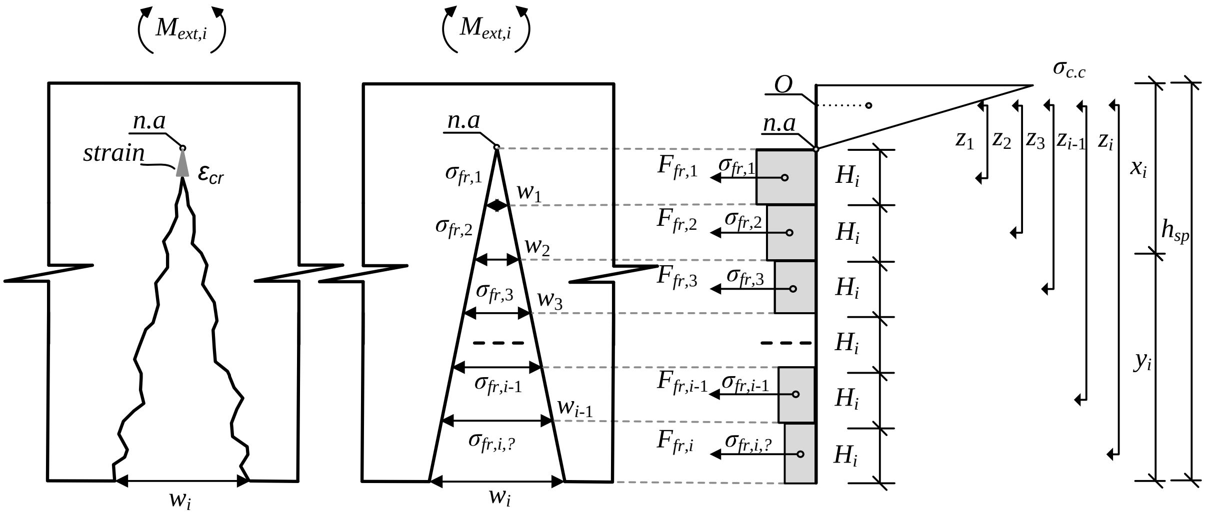


Figure 2. Analysis of the experimental diagrams according to the specified crack width intervals



Actual crack *Idealized crack* *Residual stresses distribution scheme*

Figure 3. Principal scheme for the constitutive (inverse) analysis at *i*-th crack opening step

2.2 *RILEM* technique

The most commonly used technique for the constitutive analysis of SFRC was proposed by *RILEM* [4]. It is based on the three-point bending test performed on notched specimens with dimensions of 150×150×600 mm. Employing the experimental data, next expressions are used to determine residual stresses:

$$\sigma_{fr1} = 0{,}675\frac{F_{R,1}\cdot L}{b\cdot h_{sp}^2}\;;\sigma_{fr2} = 0{,}555\frac{F_{R,2}\cdot L}{b\cdot h_{sp}^2}. \tag{6}$$

Here b is the width of the specimen; h_{sp} is the distance between tip of notch and top of cross section; L is the loading span, $F_{R,k}$ is the load recorded at the k-th stage. As shown in Fig. 4, the residual stresses σ_{fr1} and σ_{fr2} are attained corresponding to deflections of 0.46 and 3 mm, or crack width of w_1=0.4 and w_2=2.86 mm.

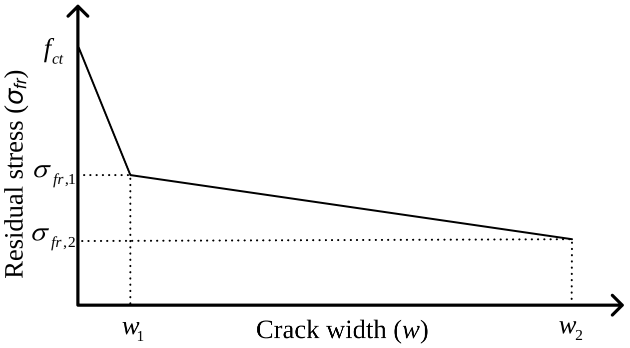


Figure 4. Residual stress-crack opening relationship by *RILEM*

2.3 Empirical method by *Naaman*

Naaman [10] proposed an alternative way to define the post-cracking behaviour of SFRC. It uses an analytical expression for residual strength that takes into account pullout length ratio and the efficiency of fibres (i.e. orientation of the fibres in respect to the cracked plane):

$$f_r = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \tau \cdot RI; \quad RI = V_f \cdot \beta \cdot \left(l_f / d_f \right), \tag{7}$$

where λ_1 is the expected pullout length ratio; λ_2 is the efficiency factor of orientation in the cracked state; λ_3 is the group reduction factor associated with the number of fibres pulling out per unit area; τ is the average bond stress of a single fibre embedded in the concrete; V_f is the steel fibre volume percentage; β is the bond factor; l_f and d_f are the length and diameter of fibre, respectively. Following *Campione* [11], the factor β can be assumed equal to 1.0 for hooked fibres (used in current study).

3 EXPERIMENTAL PROGRAM

This section reports test results of two series of SFRC beams. The series were made of concrete with different dosages of the hooked-end steel fibres: 40 and 80 kg/m^3. Main parameters of the test specimens are listed in Table 1, where $f_{cm,cube}$ is the concrete compressive strength measured at the age of 28 days on 150 mm cubes, V_f is the fibre content by volume in the concrete mixture, ρ_f is the fibre dosage by weight; l_f is the length of fibre, d_f is the diameter of fibre, *h*, *b*, *L* are the height, width and length of specimens, respectively.

Each beam was equipped with a notch of 25 mm at the mid-span. The presence of the notch localizes cracking process and allows measuring the opening crack width. However, it should be noted, that the notch ensures cracking of the beam at the place that is not necessarily the weakest section of the specimen.

Table 1. Characteristics of the specimens

Specimens series	h	b	L (span)	l_f	d_f	$f_{cm,cube}$	ρ_f	V_f
	mm					MPa	kg/m^3	%
S1	150	150	600 (500)	50	1	44,8	80	1,0
S2	150	150	600 (500)	50	1	43,1	40	0,5

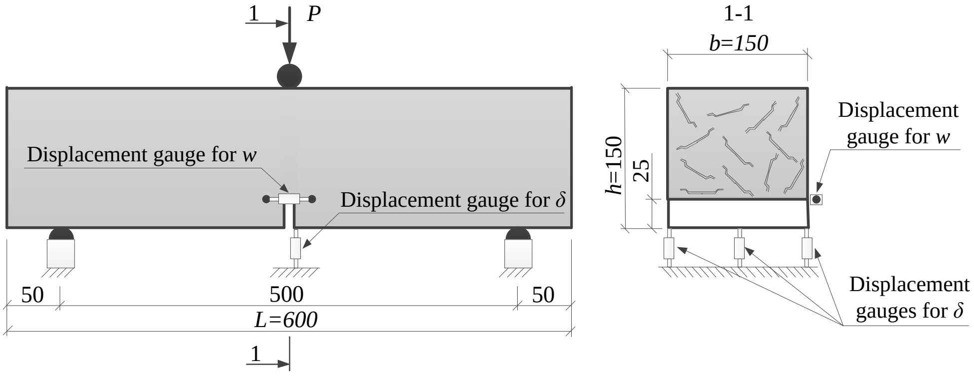
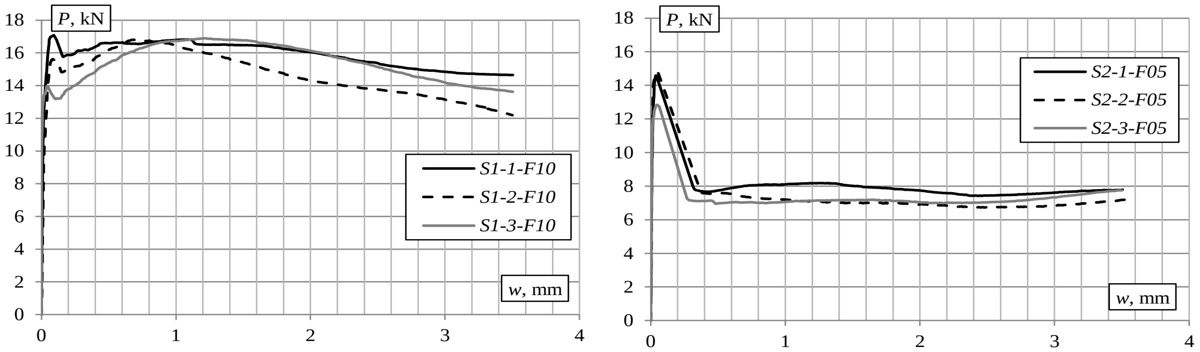


Figure 5. Experimental set-up of the beam



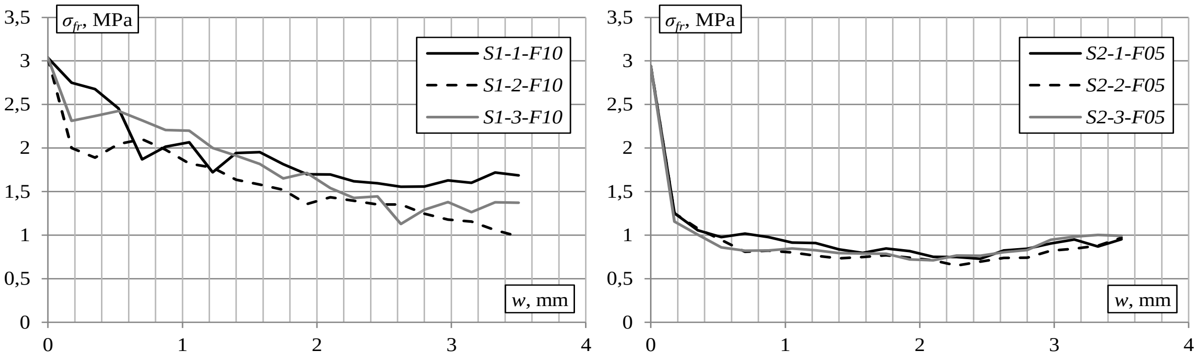
a) b)

Figure 6. Load-crack width relationships derived for the beams with 1.0% (a) and 0.5% (b) fibre volumes

The beams were loaded under three-point scheme according to *RILEM* [4] (Fig. 5). During the test, the mid-span deflections and crack opening width were measured using displacement gauges (linear variable displacement transducers). The tests were performed under displacement control with a rate of 0.2 mm/min. Employing the test results of the individual beams, load-crack width curves have been constructed (Fig. 6).

4 CONSTITUTIVE MODELLING RESULTS

The proposed inverse technique (Section 2.1) was applied to the experimental data described in Section 3. The obtained residual stress-crack opening width diagrams are given in Figs. 7a and 7b for the beams with 1.0 and 0.5% of fibres by volume, respectively.



a) b)

Figure 7. Residual stress-crack opening relationships for the beams with 1.0% (a) and 0.5% (b) fibre volumes

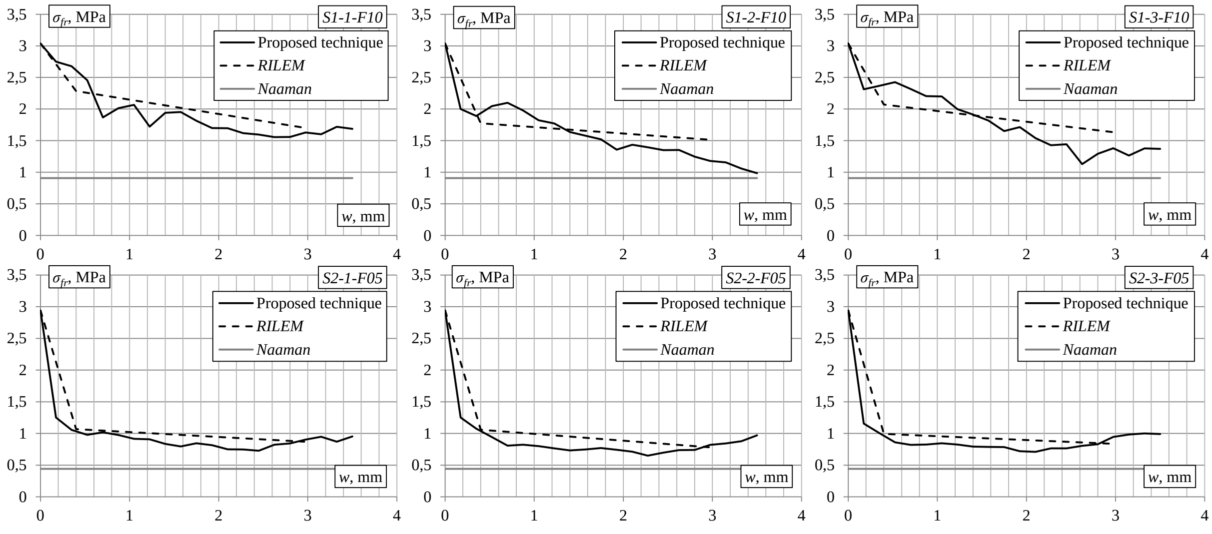


Figure 8. Residual stress-crack opening relationships

In Fig. 8, the residual stress-crack opening width relationships derived by the inverse technique are compared with the constitutive modelling results obtained by the *RILEM* [4] and *Naaman* [10] methods. It can be observed that the diagram obtained by the *RILEM* method is represented by a bi-linear curve, while *Naaman*'s approach gives constant value of the residual stresses. On the contrary, using the proposed technique, the residual stresses can be found at any loading steps that results in a fully specified material model of SFRC in tension.

5 NUMERICAL MODELLING

This section comprises verification of constitutive modelling results, discussed in Section 4. The results obtained by the inverse technique, *RILEM* and *Naaman* methods were implemented in nonlinear finite element analysis program *ATENA* as material law of SFRC in tension. The software allows obtaining *P-w* relationship for a given post-cracking behaviour of SFRC member, described by σ_{fr}-w curve. Simulated *P-w* relationships were compared with the experimental data and are given in Figs. 9 and 10. It can be observed, that the methods proposed by the authors and by *RILEM* are quite adequate and have the prediction errors lesser than 10%, while *Naaman* methods gives an error about 65%.

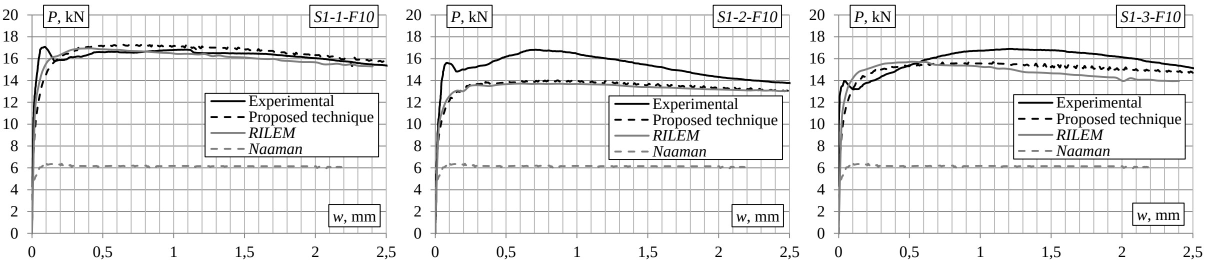


Fig.9. Comparison of experimental and calculated *P-w* diagrams of the test beams with 1.0% volume of fibres

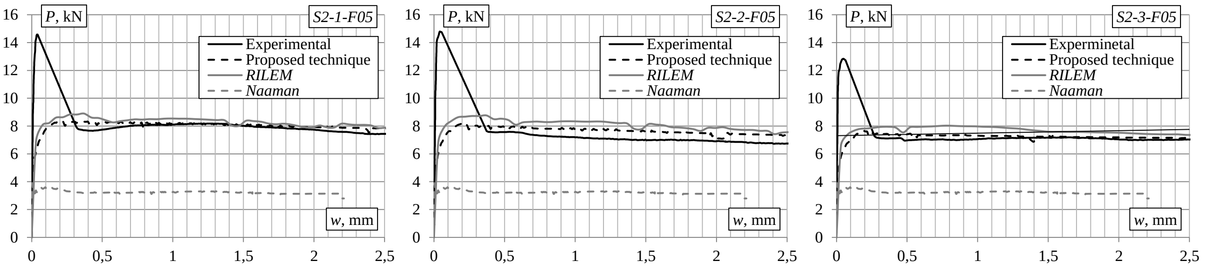


Fig.10. Comparison of experimental and calculated *P-w* diagrams of the test beams with 0.5% volume of fibres

6 CONCLUSIONS

The present study investigates the post-cracking behaviour of steel fibre reinforced concrete (SFRC) beams. Six notched beams containing two different fibre contents (0.5 and 1.0% by volume) were tested under a three-point scheme of loading. Considering SFRC as a homogeneous material, the constitutive (inverse) analysis technique for determination of the residual stresses of flexural SFRC member was proposed. The main advantage of the proposed technique (with respect to *RILEM* approach limited by standard small specimens) is its capability of deriving residual stresses on a basis of any scale flexural element test results.

Adequacy of the proposed method was verified using nonlinear finite element analysis program *ATENA*. The finite element simulation results indicated that the empirical method given by *Naaman* underestimates the experimental results and leads to an error up to 65%. The proposed and RILEM techniques had similar adequacy, while investigating the standard bending specimens. Further research should be performed studying SFRC specimens of different sizes.

REFERENCES

[1] ACI 544–1R, State-of-the-art report on fiber reinforced concrete, Technical Report, American Concrete Institute, 2002.

[2] Gribniak, V. , Kaklauskas, G. , Kwan, A. K. H. , Bačinksas, D. , Ulbinas, D. (2012), "Deriving stress-strain relationships for steel fibre concrete in tension from tests of beams with ordinary reinforcement", Engineering Structures 42, pp. 387-395.

[3] Vandewalle, L. (2000). Cracking behaviour of concrete beams reinforced with a combination of ordinary reinforcement and steel fibers, Materials and Structures 33, pp. 164-170.

[4] RILEM TC 162-TDF (2002). Test and design methods for steel fibre reinforced concrete - Design of steel fibre reinforced concrete using the σ-w method: principles and applications. *Materials and Structures*, 35: 262-278.

[5] EN 14651 (2005). Test method for metallic fibered concrete – Measuring the flexural tensile strength (limit of proportionality (LOP), residual). European Committee for Standardization, Brussels.

[6] DBV (German Concrete and Construction Technology Association) (2001). DBV-steel fiber reinforced concrete. Deutscher Beton und Bautechnik-Verein EV, Germany.

[7] DAfStb, Deutscher Ausschuß für Stahlbeton: Richtlinie Stahlfaserbeton (DAfStb, German Committee for Structural Concrete: technical rule on steel fibre concrete), Germany 2010 [in German].

[8] Kaklauskas, G., Gribniak, V., Bačinskas, D. (2011). Inverse technique for deformational analysis of concrete beams with ordinary reinforcement and steel fibers, Procedia Engineering 14, pp. 1439-1446.

[9] di Prisco, M., Plizzari, G., Vandewalle, L. (2009). Fibre reinforced concrete: new design perspectives, Materials and Structures 42, pp. 1261-1281.

[10] Naaman, A. E. (2003). Engineered Steel Fibers with Optimal Properties for Reinforcement of Cement Composites. *Journal of Advanced Concrete Technology*, 1(3): 241–252.

[11] Campione, G. (2008). Simplified flexural response of steel fiber reinforced concrete beams. ASCE Journal of Materials in Civil Engineering, 20(4): 283-293.