# EFFECT OF PARTIAL THROUGH CRACK DEPTH SIZE ON STRENGTH OF SHELL STRUCTURES

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## ABSTRACT

*The analysis of numerous cases of failure in shell structures has found to occur at stresses lower than the design stresses. The origin of these failures has been due to flaws or cracks. In order to determine which depth size of crack is admissible, one must study how the structural strength is affected by crack depth.*

*In this paper the analysis is done using finite Element program developed by the authors. The stress Intensity factor Ks of the shell structure is calculated and compared with that of theoretical one and excellent results were achieved as shown in the figures and tables of the results. The load carrying capacity of cracked shell structures were also evaluated and compared with that of the uncracked shell structures capacity to determine the crack size that is admissible. The results obtained were also compared with that of theoretical solutions and excellent results were achieved as shown in the figures (10,11,12 & 13). The elements used in this program are 8-nodes shell element; 6-nodes singular triangular shell element and 8-nodes transition singular shell element, all elements with 5-degrees of freedom per node.*

## INTRODUCTION

Any structure contains inherent flaws and cracks. They exists either originally in the material of structure during its manufacture or may develop due to repeated or sustained loads. These cracks usually starting from a flow and grow slowly in size until they become critical. Due to the presence of such cracks, the stresses will reach high local values especially at the crack-tip where it becomes infinite (i.e., singular). Cracks impair the strength of the structure. Thus, during the continuing development of cracks, the structural strength decreases until it becomes so low that the service load cannot be carried anymore, and fracture ensues.
The fracture will be in a form of unstable crack propagation, and the failure will be of catastrophic nature.

A large number of fracture failures were reported during the last century. Most of them were noticed to be shell structures like oil tanks, pressure vessels, bridges airplanes and nuclear power structures. If fracture is to be prevented, the strength of the shell should not allowed to be dropped below a certain safe value. This means that cracks must be prevented from growing to a size at which the strength would drop below an acceptable limit. In order to determine which size of crack is admissible, one must be able to study how to control crack propagation and how the structural strength is affected by crack size. Crack propagation is controlled by single factor called stress intensity factor "K".

## ANALYTIC DETERMINATION OF STRESS INTENSITY FACTORS (SIF)

The amplitude of the stress (near the crack tip) could be represented by a single parameter termed the stress intensity factor (K), which is responsible to reflect the effects of geometry of the cracked body, configuration of the crack and the applied loads at the boundaries. One of the basic principles of fracture mechanics is that the unstable crack propagation leading to brittle fracture occurs when the intensity factor reaches a critical value ($K_c$) known as fracture toughness of the material which can be obtained experimentally.
Values of ($K_c$) are available in literature for different engineering materials (Hellan 1985)andAyhan and Nied (2002 ). Many researchers dealt with stress field near the crack tip analytically. Folias (1995) in his paper showed that the stresses at the crack tip for shells are of two components; The extential stress component is:

$$\begin{Bmatrix} \sigma_x^e \\ \sigma_y^e \\ \tau_{xy}^e \end{Bmatrix} = A_e \frac{k_p}{\sqrt{2\pi r}} \begin{Bmatrix} \frac{3}{4}\cos\left(\frac{\theta}{2}\right) + \frac{1}{4}\cos\left(\frac{5\theta}{2}\right) \\ \frac{5}{4}\cos\left(\frac{\theta}{2}\right) - \frac{1}{4}\cos\left(\frac{5\theta}{2}\right) \\ -\frac{1}{4\sin}\left(\frac{\theta}{2}\right) + \frac{1}{4}\sin\left(\frac{5\theta}{2}\right) \end{Bmatrix} \tag{1}$$

And the bending stress component is:

$$\begin{Bmatrix} \sigma_x^b \\ \sigma_y^b \\ \tau_{xy}^b \end{Bmatrix} = A_b \frac{k_p}{\sqrt{2\pi r}} \begin{Bmatrix} \frac{3-3v}{4(3+v)}\cos\left(\frac{\theta}{2}\right) - \frac{1-v}{4(3+v)}\cos\left(\frac{5\theta}{2}\right) \\ \frac{11+5v}{4(3-v)}\cos\left(\frac{\theta}{2}\right) + \frac{1-v}{4(3+v)}\cos\left(\frac{5\theta}{2}\right) \\ -\frac{7+v}{4(3+v)}\sin\left(\frac{\theta}{2}\right) - \frac{1-v}{4(3+v)}\sin\left(\frac{5\theta}{2}\right) \end{Bmatrix} \tag{2}$$

*Where:*
r, θ: polar coordinates with origin at crack-tip.
$\sigma_x^e, \sigma_y^e, \tau_{xy}^e$ : Parallel, perpendicular to crack prolongation extensional stresses and shear stress.
$\sigma_x^b, \sigma_y^b, \tau_{xy}^b$ : parallel, perpendicular to crack prolongation bending stresses and shear stress.
$A_a$ ,$A_b$ : membrane and bending components of shell stress intensity factor.
$K_p$: the stress intensity factor for the corresponding flat plate = $\sigma\sqrt{\pi a}$ .
Folias gave formulas for calculating $A_e$ and $A_b$ which are dependent on crack shell parameter λ defined by:

$$\lambda = \left[12\left(1-v^2\right)\right]^{1/4} * a/[Rt]^{1/2} \tag{3}$$

Where: R is the radius of curvature.
Erdogan and Kibler (1969) presented an equation for $K_s$:

$$K_s = (A_e + A_b)\ K_p \tag{4}$$

Where: $K_p$ is the SIF in the corresponding flat plate.
The value of $K_p$ depends on the type of applied load and calculated as:
For stretching loads:

$$K_p = \left(N_0/t\right)\sqrt{\pi a} \tag{5}$$

Where: a half crack length
For bending loads:

$$K_p = \left(6M_0/t^2\right)\sqrt{\pi a} \tag{6}$$

Where:
$N_0$ : uniformly distributed applied load.
$M_0$ : uniformly distributed applied moment.
For pipes under internal pressure:

$$K_P = \left(PR/t\right)\sqrt{\pi a} \tag{7}$$

Where:
$P$ : internal pressure.
$R$ : radius of the pipe.
$t$ : thickness of pipe.

**FINITE ELEMENTS USED IN THE ANALYSIS**

The crack tip is modelled by one row of singular elements. The singular elements used are the 6-noded collapsed triangular isoperimetric quarter point element (Barsoum 1977), see Fig. (1). Transition element is used to surround the raw of singular elements. This transition element of 8-noded quadrilateral quadratic isoperimetric singular element (Lynn and Ingraffea 1978). See Fig. (2). The rest of elements used far from crack tip are 8-noded quadrilateral quadratic isoperimetric curved shell elements. See Fig. (3). The SIF is evaluated from displacements rather than stresses, because the finite elements used are based on assumed displacement fields (Ayhan and Nied 2002). Therefore, they estimate the displacement better than stresses. The procedure adopted is based on equating the terms containing the $\left(\sqrt{r}\right)$ in both analytical and finite element displacement expansions.
The collapsed isoperimetric quarter point triangular element can represent a radial displacement variation through the element of the form:

$$u_j = a_j + b_j\sqrt{r} + c_j r \tag{8}$$

Where: j = x or y.
$a_j$ ,$b_j$ and $c_j$ are constants.

Along the element edge, the nodal point displacement $u_j(0)$, $u_j(L/4)$ and $u_j(L)$ could be substituted into equation (8) and solving the resulting three simultaneous equations to find the constants $a_j$ , $b_j$ and $c_j$ as:

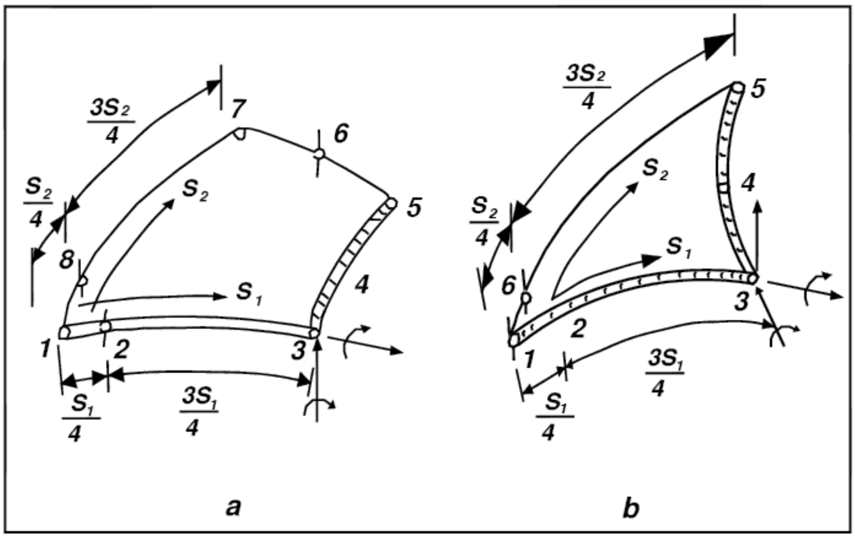


Figure (1): a) crack-tip singular quadrilateral general shell element, b) crack-tip singular triangular general shell element.

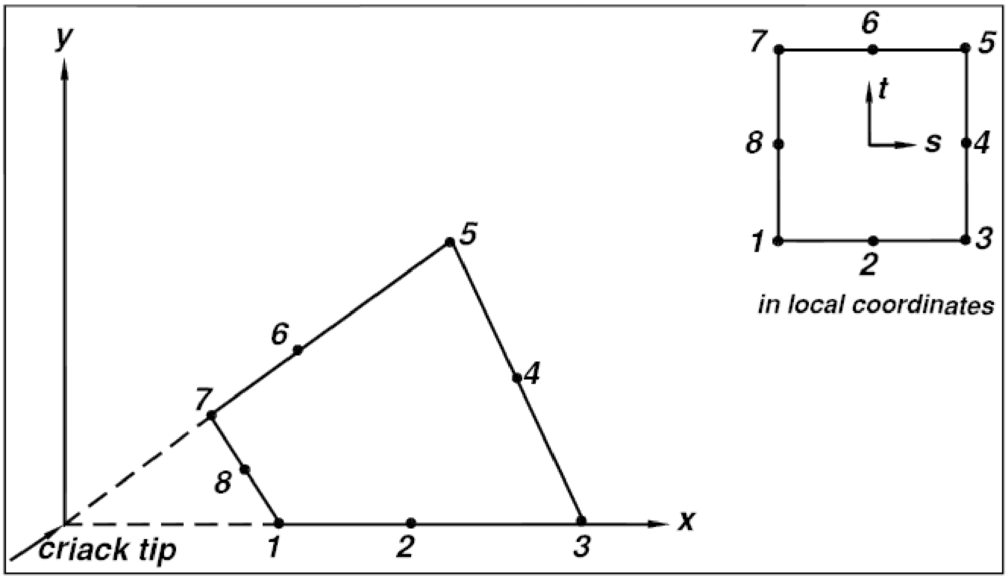

Figure (2): Eight nodded  transition element.

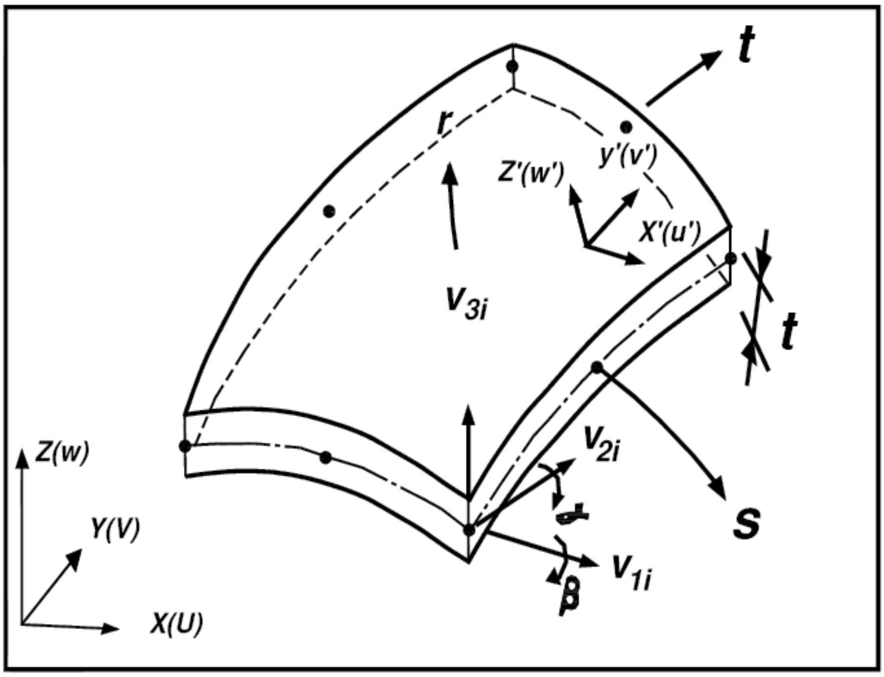

Figure (3): 8-noded curved shell element with local and global coordinates.

$$
\begin{aligned}
a_j &= u_{j(0)} \\
b_j &= \left( 1 \big/ \sqrt{L} \right) \left[ 4u_{j\left(L/4\right)} - u_{j(L)} - 3u_{j(0)} \right] \\
c_j &= \left( 1 \big/ L \right) \left[ 2u_{j(L)} + 2u_{j(0)} - 4u_{j\left(L/4\right)} \right]
\end{aligned} \tag{9}
$$

Substituting the constants' values of eqn. (9) into eqn. (8) yields:

$$
u_{j(r)} = u_{j(0)} + \left[ 4u_{j\left(\frac{L}{4}\right)} - u_{j(L)} - 3u_{j(0)} \right] \sqrt{r/L} + \left[ 2u_{j(L)} + 2u_{j(0)} - 4u_{j\left(\frac{L}{4}\right)} \right] \frac{r}{L} \tag{10}
$$

Where:
$u_x$ = u-displacement in x-direction.
$u_y$ = v-displacement in y-direction.
The analytic expression of the displacement along any radial line emanating from the crack tip can be expressed in general form as:

$$u_{j(r)} + B_j\sqrt{r} + o_{(r)} \tag{11}$$

Where:
j = x or y.
$B_j$ is defined as:

$$B_j = \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} = \frac{K_{PE}}{2G\sqrt{2\pi}} \begin{Bmatrix} \cos\left(\frac{\theta}{2}\right)\left[k-1+2\sin^2\left(\frac{\theta}{2}\right)\right] \\ \sin\left(\frac{\theta}{2}\right)\left[k+1-2\cos^2\left(\frac{\theta}{2}\right)\right] \end{Bmatrix} \tag{12}$$

Comparing eqn. (11) with eqn. (10) one concludes that:

$$B_j = \left( \frac{1}{\sqrt{L}} \right)\left[ 4u_{j\left(L/4\right)} - u_{j(L)} - 3u_{j(0)} \right] \tag{13}$$

Substituting eqn. (13) into eqn. (12) and considering only vertical displacement component, one gets:

$$K_S = \frac{2G\sqrt{2\pi}}{\sqrt{L}} \left[ \frac{4v_{mid\left(L/4\right)} - v_{mid(L)} - 3v_{mid(0)} + 0.5r\left( 4t_{\left(L/4\right)}\alpha_{\left(L/4\right)} - t_{(L)}\alpha_{(L)} - 3t_{(0)}\alpha_{(0)} \right)}{\sin\left(\theta/2\right)\left[k+1-2\cos^2\left(\theta/2\right)\right]} \right] \tag{14}$$

The subscripts (0), (L/4) and (L) are used to denote the displacement at the crack-tip node, the quarter point node and the edge node respectively.

The values of ($k_s$) are evaluated at both sides of each triangular quarter point that are used around the crack-tip. This is done by substituting the (□) value for each ray (which represents a side of two adjacent singular elements) into eqn. (14) to get a value of SIF at each ray. The average value approach has been adopted in this study (Proudhon and Besseville).

## DETERMINATION OF LOAD CARRYING CAPACITY FOR UNCRACKED SHELLS:

The target of present study is to show the effect of partial through cracks on the strength of shell structures and how does the load carrying capacity of original structures decreases when cracks exist. Therefore, one must estimate the failure loads of uncracked shell structures before estimating the failure loads of the cracked shell structures. Evaluation of the failure load of uncracked structure requires knowledge of yield criterion that will be used.

## YIELD CRITERIA:

In practice, most materials deform plastically once some critical combination of stresses is achieved. The yield criterion adopted in this paper is Von Mises Criterion which predicts that yielding will occure when shear strain energy per unit volume reaches a critical value and given by:

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2 = \sigma_y^2 \tag{15}$$

For plane strain problems $\tau_{yz} = \tau_{zx} = 0$ , then eqn. (15) will be:

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\tau_{xy}^2 = \sigma_y^2 \qquad (16)$$

The square root of the left hand side of eqn. (16) may be called the Von Mises stress. The load which makes the Von Mises stress at critical point or points in a bearing structure reach the uniaxial yield stress of its material is considered as the failure load of the uncracked state of structure. The global stress components in the left hand side of equations (15,and 16) were evaluated at the Gauss integration points of the finite elements because there the stresses will be most accurate than everywhere else inside the element domain including the nodes.

After calculating stress components at each Gauss point, the square root of their combination according to equations (15, 16) is taken and turned as Von Mises stress. The procedure is that the applied load is increased gradually until this Von Mises stress reaches the uniaxial yield strength of the material ($\square_y$). Then, this applied load is the "carrying capacity of uncracked shell structure". The material used in this study is "AISI-4340 steel", since it well studied and tested due to its practical importance in aircraft industry and its properties are listed in Table (1) (Putatunda 1996). In this Table, $\square_y$ = 1014MPa. Thus, the applied load which makes the Von Mises stress at any Gauss point in the problem model reach the value (1014MPa) is considered as the load carrying capacity of uncracked case of the problem. The value of fracture toughness ($K_s$) for "AISI-4340 steel" adopted in this study is 6000MPa $\sqrt{mm}$ .

**APPLICATIONS:**

Large number of cracked shells contains such cracks .for example the flaws that occur in pipes, gun barrels and tanks are of this type. Such problems are classified according to the way they are modeled into. Among these problems is the hollow cylinders under high pressure .The classical solution for this structure was developed by Lame( Timoshenko,1970 ) shows that the largest stress is the tangential stress that occur at inner surface which initiate a radial cracks. Figure (4) shows all possible types of cracks may happen. This problem is modeled as a plain strain problem and will be studied.

**PROBLEM-1:**

The problem considered here is the analysis of: “A hollow cylinder containing a radial crack emanating from the inner or outer surface and subjected to internal pressure, See fig. (4). The following problem was studied to find its load carrying capacity. In this problem the cylinder has one radial crack emanating from the outer surface, as shown in Fig.(4. c). This Problem has symmetry only about X-axis ,therefore half of the cylinder cross section should be modeled in finite element meshes. The finite element meshes used in the analysis have their characteristic written in table(2) and illustrated in Fig.(5 ). First, the finite element solution for the uncracked case of the problem for the radial displacement of inner surface was solved and compared with the theoretical one calculated from the so-called “Lame Stresses “ given by (Timoshenko & Goodier 1972) as follows :

Lame Stresses:-

$$\sigma_r = \frac{a^2 P}{b^2 - a^2}\left[1 - \frac{b^2}{r^2}\right].$$

$$\sigma_\theta = \frac{a^2 p}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \qquad (17)$$

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) = \frac{2\nu a^2 P}{b^2 - a^2}. \qquad \text{and} \qquad \tau_{r\theta} = 0 .$$

Where :

ν: Poisson's ratio.

a,b : Inner and outer radial respectively.

P : Internal Pressure.

Hoop strain ($\varepsilon_\theta$)defined by ( Timoshenko & Goodier,1970 ) as :

$$\varepsilon_\theta = \frac{u}{r} = [\frac{\sigma\theta}{E} - \nu \ (\frac{\sigma_r}{E}) - \nu\ (\frac{\sigma z}{E})]. \quad (18).$$

Substituting Eqn's.( 17 )into Eqn. (18 ) one gets the theoretical radial displacement as:

$$u = \frac{a^2 p(1+\nu)}{E(b^2 - a^2)}\left[(1-2\nu)r + \frac{b^2}{r^2}\right]. \quad (19).$$

Using the material properties of “AISI-4340 steel from table ( 1 ) and geometry from Fig.( 4 ) , the radial displacement ( u ) from Eqn.(19 ) is equal to : $u_{theo.}$= (9.079365 * $10^{-5}$ P ),whereas the finite element solution gives : $u_{num.}$= (9.0785 *$10^{-5}$ P ).The error % is 0.01%. The failure internal pressure which makes the Von Mises stress equal to( $\sigma_y$) of the material can be derived theoretically following Eqn. ( 16 ) as follows :

$$\text{Von Misses Stress}=\sqrt[2]{\sigma^2 r + \sigma^2 \theta + \sigma^2 z - \sigma_r\sigma\theta - \sigma\theta\sigma z - \sigma z\sigma_r} \quad (20).$$

Substituting the stresses of Eqn's (17 ) into Eqn. (20 ) yields :

$$\text{Von Misses Stress}= \frac{a^2 P\sqrt{(1-2\nu)^2 + 3(\frac{b^4}{r^4})}}{(b^2 - a^2)} \quad (21).$$

Applying the geometry data of the problem from Fig. (4. a) and equating Von Mises stress with ($\sigma_y$= 1014 MPa ) the theoretical value of the failure internal will be :$(P_{fu})_{theo.}$ = 438 MPa , whereas from the finite element solution ; :$(P_{fu})_{num.}$ = 460 MPa, the % error between the two values is ( 4.94 % ).

The values of S.I.F. from the finite element solution and the theoretical ones done by(Tracy,1979) for this problem for each (c/t ) value together with the % error between them are listed in tables ( 3 )and graphed in figure ( 6) .

The values of load carring capacity (which are the values of internal pressure that make the S.I.F. reach its critical value ;($K_{IC}$ = 3358 MPa$\sqrt{mm}$ ) resulted from the finite element solution and those obtained from the analytical solution with the comparison between them at each ( c/t )value are tabulated in tables ( 4) and illustrated in figures ( 7).

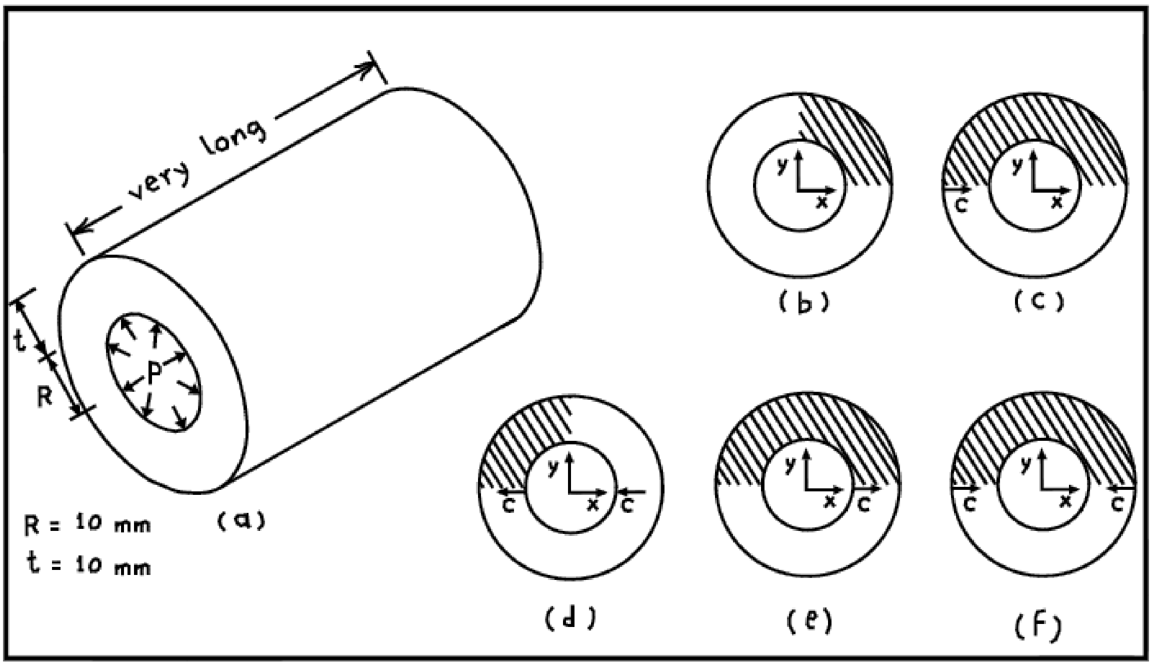


Figure. (4): Details of problem (1). (a):Geometry and dimension. (b): uncracked state (c) prob. ( 1 ). ( d ,e, and f) are for other possibilities of crack initiations.

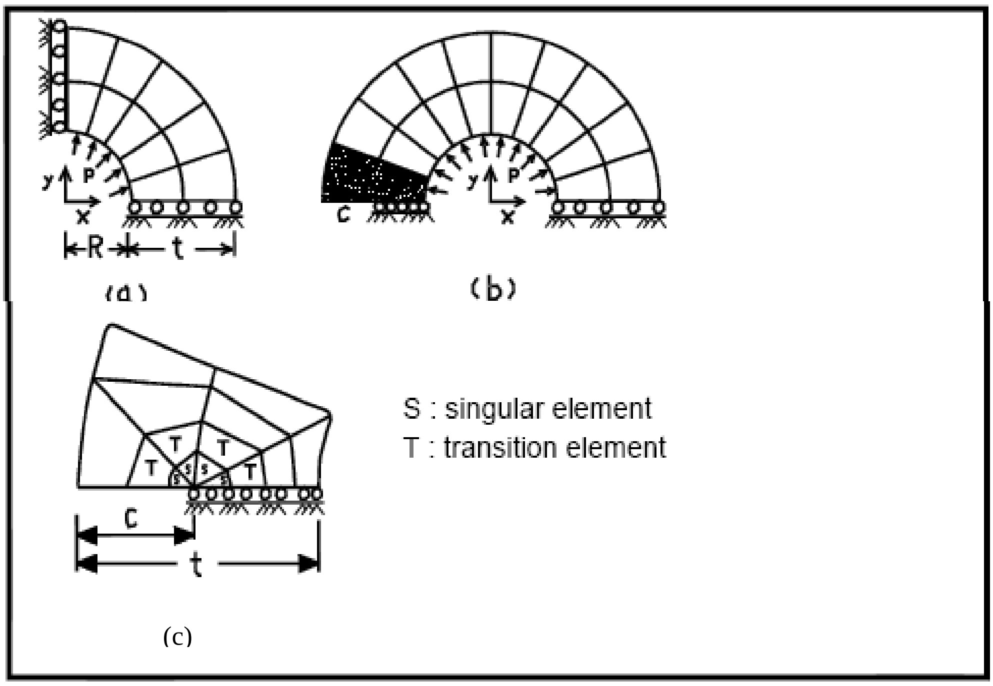


Figure (5) : Finite element meshes : (a) : uncracked state. (b): cracked state. ( c ) :details of dark zone.

Table -3 : Non-dimensional S.I.F. vs.non-dimensional crack depth for problem (1)

| c/t | $K_{PE}/P\sqrt{\pi c}$ | | % error |
|---|---|---|---|
| | F.E. (Present Work) | Theoretical Tracy . | |
| 0.1 | | 0.800 | |
| 0.2 | 0.901 | 0.913 | 1.35 |
| 0.3 | | 1.040 | |
| 0.4 | 1.190 | 1.200 | 0.83 |
| 0.5 | 1.383 | 1.400 | 1.21 |
| 0.6 | 1.635 | 1.660 | 1.50 |
| 0.8 | 2.420 | | |

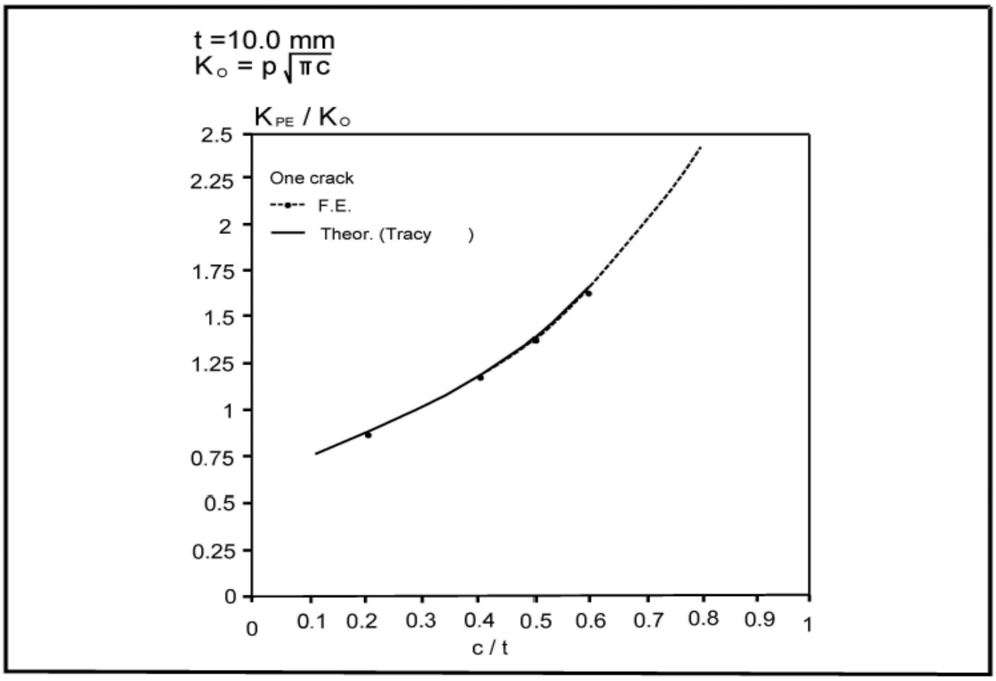


Figure. (6): Non-dimensional S.I.F. vs. non-dimensional cracked depth for problem ( 1 ).

Table – 4 :Non-dimensional failure internal pressure vs. non-dimensional cracked depth for problem ( 1 – A ).

| c/t | $P_{fc}$ / $P_{fu}$ | | ror |
|---|---|---|---|
| | F.E. (Present Work) | Theoretical (Tracy ) | |
| 0.1 | | 5.444 | |
| 0.2 | 3.233 | 3.373 | 4.10 |
| 0.3 | | 2.418 | |
| 0.4 | 1.732 | 1.816 | 4.60 |
| 0.5 | 1.333 | 1.392 | 4.20 |
| 0.6 | 1.028 | 1.071 | 4.00 |
| 0.8 | 0.602 | | |
| 1.0 | 0.103 | | |

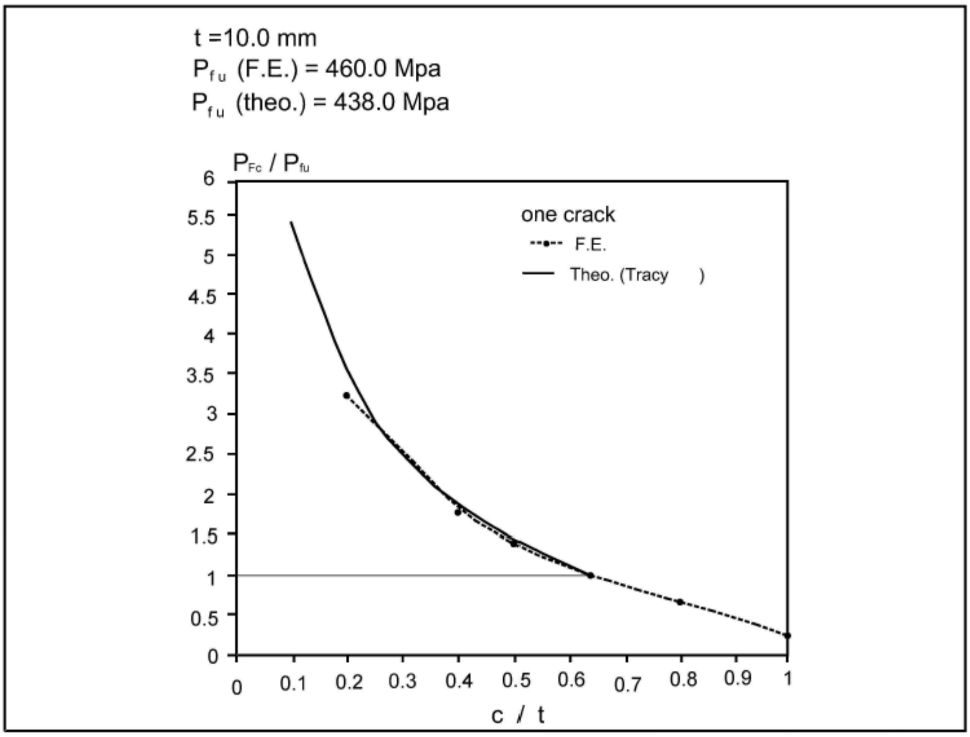

Figure. (7 ): Non-Dimensional failure internal pressure vs. non-dimensional cracked depth for problem (1).

**DISCUSSION & CONCLUSIONS:**

It is important to be sure that the calculated stress intensity factor of shell structures $K^{s}$ is correct. Figs.(6) showed that the non-dimensional stress intensity factor calculated by the developed program vs. non-dimensional crack depth was close to that calculated by the theoretical method. In Figs (7) which plot the ratios of load carrying capacity for cracked states to that for uncracked states vs. the non-dimensional cracked depth, the ratios were greater than 1 for some small crack depth ratios. That means, that the amount of load causing unstable crack propagation is greater than the one producing yielding, hence, failure in these cases are due to yielding before the unstable propagation of crack occur.

It is clear that the point of intersection of the horizontal line at ratio equals 1, with falling curve indicates the crack depth beyond which cracks will be dangerous and the load carrying capacity of the cracked structure begins to be less than that of uncracked structures. Thus, it should be indicated by fracture criterions rather than yield criterions. If failure happened with these cracks it will be in the form of unstable crack propagation, which is usually sudden, rapid and catastrophic.

Also, Figs (7) showed the role of crack depth in impairing structural strength of shells. These graphs can be used to give the maximum admissible crack depth beyond which fracture will occur.

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