DUCTILITY SPECTRUM METHOD TO ESTIMATE SEISMIC DEMANDS FOR STRUCTURES

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ABSTRACT

In this paper, an improved procedure (Ductility Spectrum Method, DSM), applicable to the analysis and design of structures is presented and illustrated by examples. This procedure uses inelastic spectra and gives peak responses consistent with those obtained when using the nonlinear time history analysis. The accuracy of the DSM method is verified against the nonlinear time history analysis using an eight-story building. The comparison showed that the DSM method is capable to furnish accurate deformations and interstory drifts.

1 INTRODUCTION

The estimation of seismic demands at performance low levels requires explicit consideration of inelastic behavior of structures. While Nonlinear Response History Analysis (NL-RHA) is the most rigorous method to calculate the seismic demands. In order to avoid complex of NL-RHA, several simple evaluation methods have been proposed. Among these methods, the Capacity Spectrum Method CSM (***ATC-40***)[1] and the Displacement Coefficient Method (***FEMA-273***)[6], such as its popularity is increasing rapidly.

The capacity spectrum method provides an overview of the inelastic behavior of structures subjected to seismic movements. However, to be more precise, it requires a realistic capacity curve of the structure that is consistent with its dynamic behavior when subjected to an earthquake. The capacity spectrum method (CSM) compares the capacity of a structure to resist lateral forces to the demands of earthquake response spectra in a graphical presentation that allows a visual evaluation of how the structure will perform when subjected to earthquake ground motion. The method is easily understandable and generally consistent with other methods that take into account the nonlinear behavior of structures subjected to strong motion earthquake ground movements (***Freeman***)[7].

In the (***FEMA-273***)[6] document, the displacement coefficient method is adopted in which the maximum inelastic deformation of a structure is estimated from the maximum linear elastic deformation of this structure by using a modifying factor. In both methods, the maximum displacement demands in buildings are computed from the results of Single-Degree-of-Freedom (SDOF) systems. Thus, estimation of the maximum displacement demands of the inelastic SDOF systems is a fundamental issue for the seismic design and evaluation of the Multi-Degree-of-Freedom (MDOF) structures.

This study presents a new method, applicable to evaluation and design of structures has been developed and illustrated by examples. This method uses inelastic spectra and gives peak responses consistent with those obtained when using the NL-RHA. Hereafter, the seismic demands assessment method is called in this paper Ductility Spectrum Method (DSM). It is used to estimate the seismic deformation of MDOF systems based on inelastic response spectrum, and it is a relatively simple method for determining the seismic demands of structures.

2 SEISMIC DEMANDS

The seismic demand in the DSM method is determined by using the Ductility Demand Response Spectrum DDRS that was developed by the author in his last work (***Chikh et al***)[2], which will be detailed in the following.

Considering an inelastic SDOF system (***Figure 1***), its motion when subjected to an earthquake ground motion is governed by the following equation:

$$m\ddot{x} + c\dot{x} + f(x,\dot{x}) = -m\ddot{u}_g(t) \quad (1)$$

Where, m, c and f represent the mass, damping, and the resisting force of the system, respectively, $\ddot{x}_g(t)$ denotes the earthquake acceleration. The resisting force f is defined as the sum of a linear part and a hysteretic part:

$$f = k_p x + Qz \quad (2)$$

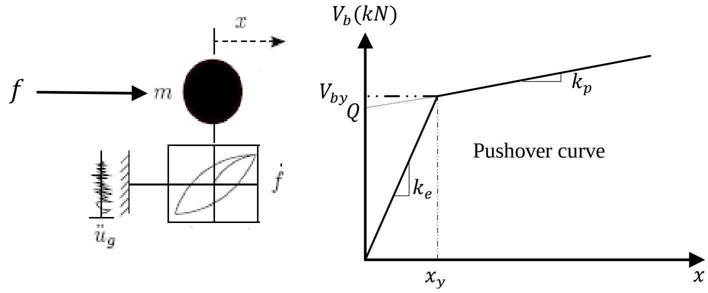


Figure 1. Capacity curve of a (SDOF) bilinear system

In the above, k_p is the postyield stiffness, Q is the yield strength, and z represents the dimensionless variable that characterizes the Bouc-Wen model of hysteresis (**Wen, 1976**)[9].

Substituting Eq. (2) into Eq. (1) and dividing by m yields:

$$\ddot{x} + 2\xi\omega\dot{x} + \alpha\omega^2 x + qgz = -\ddot{u}_g(t) \quad (3)$$

In which ξ, ω, α and q represent the damping ratio, circular frequency, post-to-preyield stiffness ratio, and the yield strength coefficient (defined as yield strength divided by the system weight W: $W = mg, g$ stands for the gravity), respectively.

Next, Eq. (3) is rewritten in terms of ductility factor, μ. Substituting: $x = x_y\,\mu$, $\dot{x} = x_y\,\dot{\mu}$, and $\ddot{x} = x_y\,\ddot{\mu}$ in Eq. (3) and dividing by x_y gives (**Chikh et al**)[2]:

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{qg}\ddot{u}_g(t) \quad (4)$$

We observe from Eq. (4) that for a given ground acceleration, $\mu(t)$ depends on ξ, ω, α and q.

To obtain meaningful system response to an ensemble of ground motions, the system yield strength coefficient has to be defined relative to the intensity of individual ground motions. Using the parameter η introduced by (**Mahin and Lin**)[8] as:

$$\eta = \frac{qg}{PGA} \quad (5)$$

Where, PGA stands for the Peak Ground Acceleration. Incorporating η into Eq. (4) results (Chikh et al, 2012):

$$\ddot{\mu} + 2\xi\omega\dot{\mu} + \alpha\omega^2\mu + \omega^2(1-\alpha)z = -\frac{\omega^2(1-\alpha)}{\eta}\overline{\ddot{u}_g}(t) \quad (6)$$

In which, $\overline{\ddot{u}_g}(t)$ represents the ground acceleration normalized with respect to the PGA.

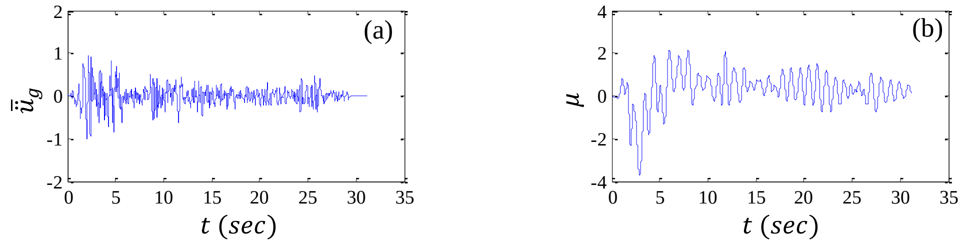


Figure 2. (a) Strong component of normalized ground acceleration of El Centro 1940 (N/S) and (b) ductility demand μ for a system with $x_y = 2.5$ cm, and $\eta = 0.25$

The ground acceleration has been normalized such that its value varies from -1 to 1 (***Figure 2a***). Eq. (6) implies that for a given inelastic system, if α and η are fixed, the intensity of the ground motion has no effect on the peak normalized deformation, μ. This permits the construction of the ductility response spectrum for an ensemble of ground motions with common frequency content but variable intensity.

- **Constant-η Ductility Demand Response Spectrum**

The procedure to construct the ductility response spectrum for inelastic systems corresponding to specified levels of normalized yield strength η, is summarized in the following steps, (***Chikh et al***) [1]:

1. Define the ground motion $\ddot{u}_g(t)$;
2. Select and fix the damping ratio ξ and the post-to-preyield stiffness ratio α ($\alpha = 0$ for for elastoplastic system) for which the spectrum is to be plotted;
3. Specify a value for η;
4. Select a value for elastic period T;
5. Determine the ductility response $\mu(t)$ of the system with, T, ξ and α equal to the values selected by solving Eq. (6). From $\mu(t)$ determine the peak ductility factor μ;
6. Repeat steps 4 and 5 for a range of T, resulting in the spectrum values for the η value specified in step 3;

The value of the ductility factor is read from the spectrum developed by the above procedure and multiplied by x_y to obtain the peak deformation, x_m.

3 CAPACITY CURVE

The governing differential equation of an MDOF system can be written as

$$M\ddot{x}(t) + C\dot{x}(t) + F\,(x, sign\dot{x}) = -M1\ddot{u}_g(t) \quad (7)$$

Where M and C are the mass and damping matrices, F denotes the story force vector, and $\ddot{u}_g(t)$ denotes the earthquake acceleration. So we can decompose movements in the form of a series of normal modes:

$$x(t) = \sum_n x(t)_n = \sum_n \phi_n q_n(t) \quad (8)$$

$q_n(t)$ Modal co-ordinate and ϕ_n is the n^{th} natural vibration mode of the structure.
The application of the force vector F over the height of the building for each time step t gives forces and stresses on the elements in a static analysis. This force distribution F can be obtained with a decomposition of the system inertia (***Chopra***)[1]:

$$M\,1 = \sum_n \Gamma_n M \phi_n = \sum_n S_n \tag{9}$$

Where Γ_n are modal participation factors and S_n modal inertia force distribution over the height of the building. If we used the orthogonality property of modes and pre-multiply equation (9) by ϕ_n^t, we obtain the following relation for the modal participation factors Γ_n:

$$\phi_n^t M 1 = \Gamma_n \phi_n^t M \phi_n \rightarrow \Gamma_n = \frac{L_n}{M_n} \tag{10}$$

And $\phi_n^t M 1 = L_n$ et $\phi_n^t M \phi_n = M_n$

Substituting Eq. (8) into Eq. (7), and using the masse and classical damping orthogonality property of modes, we obtain the following differential equation for the response of the SDOF system :

$$\ddot{q}_n(t) + 2\xi_n \omega_n \dot{q}_n(t) + F_{sn}(q_n, sing\ \dot{q}_n) = -\Gamma_n \ddot{u}_g(t) \tag{11}$$

$$F_{sn} = \frac{\phi_n^T F(q, sign\dot{q})}{M_n}$$

The resisting force depends on all modal co-ordinates $q_n(t)$, implying coupling of modal co-ordinates because of yielding of the structure. In which ω_n is the natural vibration frequency and ξ_n is the damping ratio for the nth mode.

The solution q_n of Equation (11) is given by

$$q_n(t) = \Gamma_n\, D_n(t) \tag{12}$$

With this approximation, the solution of Equation (11) can be expressed by Equation (12), where $D_n(t)$ is governed by

$$\ddot{D}_n(t) + 2\xi_n \omega_n \dot{D}_n(t) + \frac{F_{sn}(D_n, sing\dot{D}_n)}{L_n} = -\ddot{u}_g(t) \tag{13}$$

The displacement of the original structure based on modal displacements gives by

$$x(t) = \sum_n \phi_n \Gamma_n D_n(t) \tag{14}$$

If one takes only the fundamental mode, the expression reduces to:

$$x(t) \cong \phi_1 \Gamma_1 D_1(t) \tag{15}$$

From this relation, the maximum roof displacement of the structure can be obtained by

$$x_t = \phi_{N,1} \Gamma_1 D_1 \rightarrow D_1 = \frac{x_t}{\phi_{N,1}\Gamma_1} \tag{16}$$

For a correspondence between base shear of Pushover curve and corresponding acceleration of an inelastic SDOF system, we can used the resisting force in terms of the acceleration and displacement of the corresponding linear system, we get:

$$F(t)_n = K\, x(t)_n = S_n A_n(t) = \omega_n^2 S_n D_n(t) \tag{17}$$

Any response quantity $r(t)$ (story drifts, internal element forces, etc.) can be expressed as

$$r_n(t) = r_n^{et} A(t) \tag{18}$$

Where r_n^{et} denotes the modal static response, the static value of r due to external forces s_n.
In this approach, the base shear V_b, can be obtained by function of static shears stress $V_{b,n}^{st}$ induced by S_n for an known time step

$$V_{b,n}^{st} = \sum_{j=1}^{N} S_{j,n} = 1^t S_n = \Gamma_n 1^t M \phi_n \rightarrow V_{b,n}^{st} = \frac{F_{sn}}{L_n} \tag{19}$$

M_n^* is the modal effective mass associated with the n[th]-mode. Finally, the base shear V_b can be approximated by:

$$V_b(t) = \sum_n V_{b,n}^{st} A_n(t) \approx V_{b,1}^{st} A_1(t) \rightarrow A_1(t) = \frac{V_b(t)}{M_n^*} \tag{20}$$

Thus, a transform expression for base shear in Pushover analysis and corresponding acceleration to an SDOF system (***Figure 3d***). The curve Acceleration – Displacement (A-D) is known by capacity diagram of structure. This curve undergoes a similar bilinear representation of the capacity diagram. This idealization is used to calculate the normalized yield strength coefficient η and the post-to-preyield ratio α, as follows:

$$q = \frac{Q}{M_n^*} \tag{21}$$

$$\eta = \frac{q}{PGA} = \frac{Q}{PGA\, M_1^*} \tag{22}$$

4 STEP-BY-STEP DUCTILITY SPECTRUM METHOD

The Ductility Spectrum Method (DSM) described next is suitable for the design of structures as well as evaluation of existent ones. This is a direct estimation of seismic demands of structures using inelastic response spectrum, it is shown as following steps (***Figure 3***):

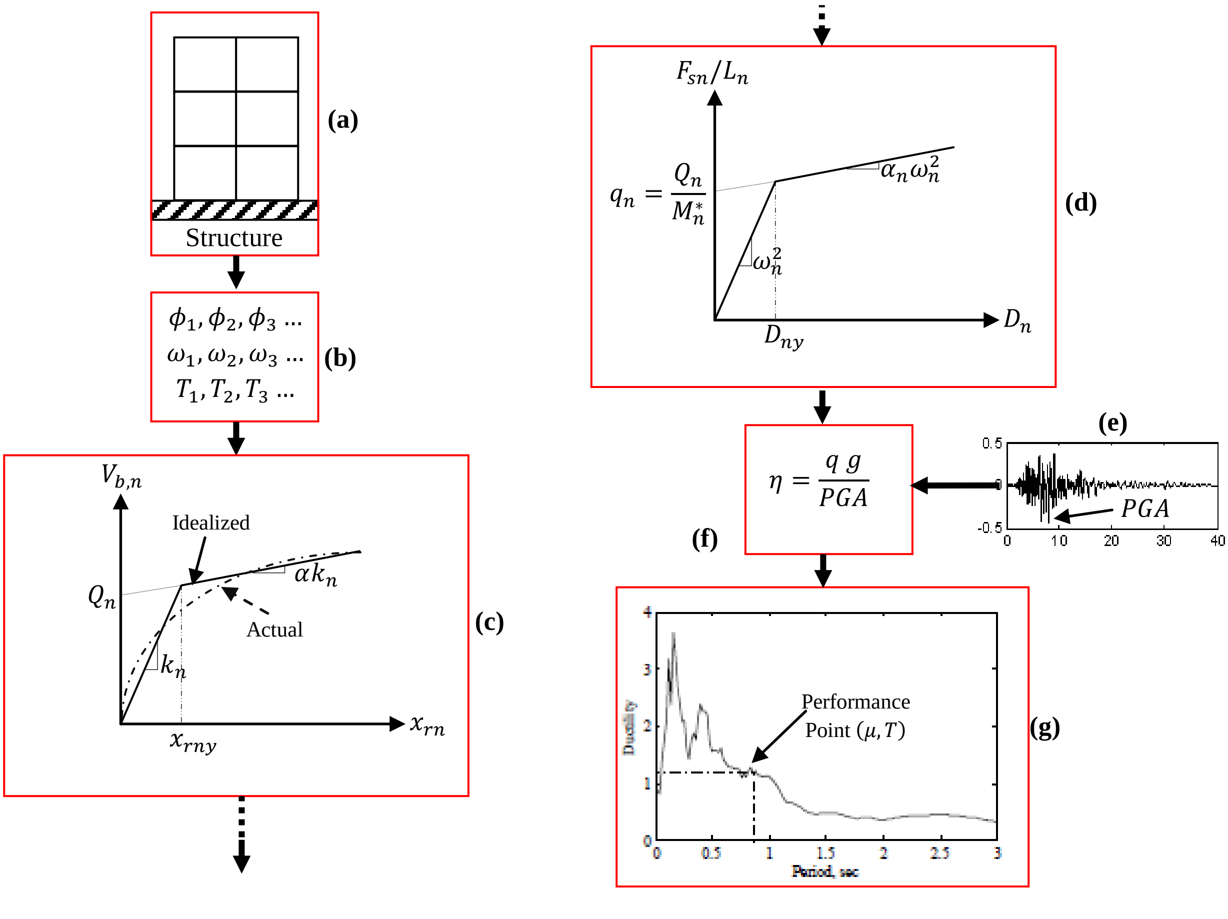
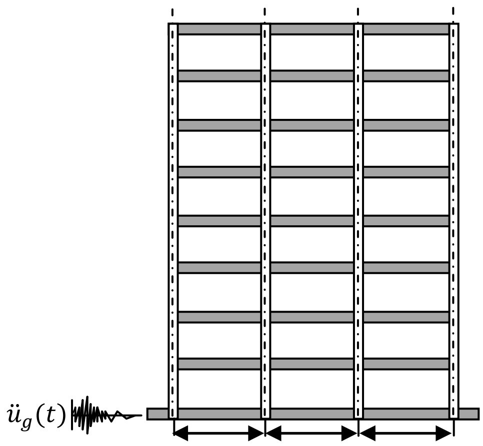


Figure 3. Flow chart of the DSM procedure

1- Calculate the natural vibrating period T_n, circular frequency ω_n and the mode shapes ϕ_n. And select a 1st mode's characteristics of the structure $(\omega_1\ and\ \phi_1)$, (***Figure 3b***).
2- Develop the diagram of force-deformation relationship between base shear and top displacement (capacity curve), this relation is obtained by nonlinear static analysis (Pushover), (***Figure 3c***).
3- Transfer the Pushover curve to the capacity diagram. Both diagrams are in the $A - D$ format (Acceleration – Displacement) using Equations (16 and 20), (***Figure 3d***).
4- Define the ground motion $\ddot{u}_g(t)$, (***Figure 3e***).
5- Compute the post-to-preyield stiffness ratio α and normalized yield strength η by Equations (22) with known q and PGA, and fix the damping ratio ξ of the design structure, (***Figure 3f***).
6- Construct the Ductility Demand Response Spectrum (DDRS) for the design earthquake (s). Graphically, draw a vertical line at T_1 on the DDRS-η and pick out the intersection points, ductility demand μ, (***Figure 3g***).
7- Calculate the maximum inelastic displacement x_m and the corresponding base shear from the capacity curve obtained in tne secod step of this procedure.

4 EXAMPLE

An eight-story reinforced concrete plane frame structure is considered in the following implementation-investigation. As shown in figure (4), the reinforced concrete frame structure consists of three-bay frame, spaced at 4 m and a story height of 3 m with no significant height irregularities. The purpose of this study is to confirm the application of the proposed method for each frame structure under a design earthquake. The vibration modes and periods of the building for linearly elastic vibration are presented in table 1. The capacity curve for the first mode is shown in Figure (5a). This curve will be transformed in the capacity diagram (A-D) format, and is shown in figure (5b).



Story	Columns (cm)	Beams (cm)
1, 2 and 3	60*60	
4, 5 and 6	50*50	40*35
7 and 8	40*40	

Figure 4. Geometric properties of the structure

This structure is subjected to the El Centro 1940 ground motion (N/S) component $(\mathrm{PGA} = 0.32\mathrm{g}, \mathrm{PGV} = 36.14\ \mathrm{cm/sec}, \text{and } \mathrm{PGD} = 21.34\mathrm{cm})$. To ensure that this structure responds well into the inelastic range, the El Centro ground motion is scaled up a factor varying from 1.0 to 2.0.

Table 1. Modal informations

Mode	$f_n\ (Hz)$	$T_n\ (sec)$	Γ_n	$M_n^*(\%)$
1	1,22	0,81	1,558	77,10
2	3,60	0,27	-0,611	11,87
3	6,40	0,15	-0,386	4,74

The seismic demands of the building is determined by the DSM method, and compared with the 'exact' results of a non-linear dynamic analysis NL-RHA using the IDARC computer program.

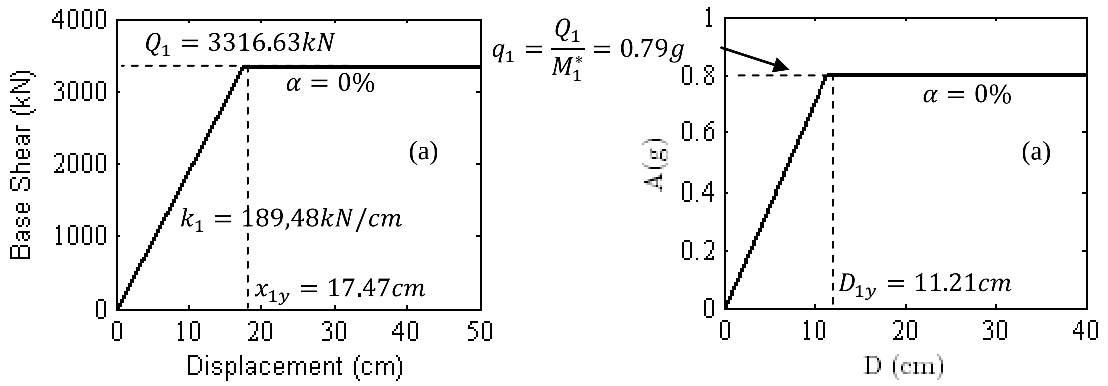


Figure 5. (a) Capacity curve and (b) capacity diagram

Figure 6 shows the DDRS-η spectrum for the El Centro 1940 (N/S) earthquake constructed assuming a normalized yield strength η as known. Graphically, with a vertical line at $T = 0.81\,sec$ on the DDRS-η, the value of the ductility factor is read from the spectrum.

Table 2. Values of η

SF	PGA	η
1	0.318	2.50
1.5	0.477	1.67
2	0.636	1.25

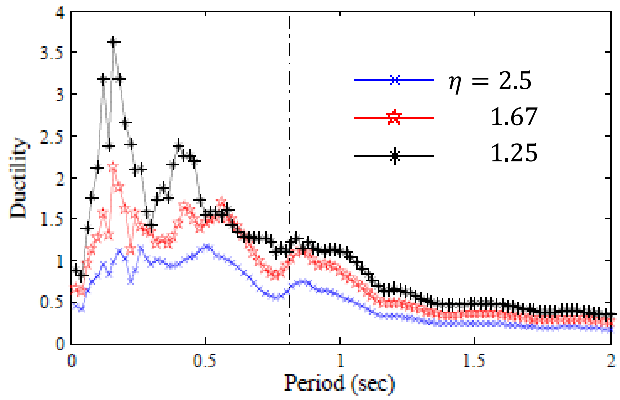


Figure 6. DDRS for El Centro 1940 (N/S) component (η = 2.5, 1.67 and 1.25)

Peak displacement profiles and inter-story drift ratio profiles estimated by the NL-RHA analyses and predictions by DSM procedure for the building are shown in Figure 7. The DSM procedure both result in similar estimates and generally yield better estimates of the peak displacement profile particularly for the story. Comparing the time-history responses for the different accelerations indicates that the difference between the ground motion generally produce more variability in the demands.

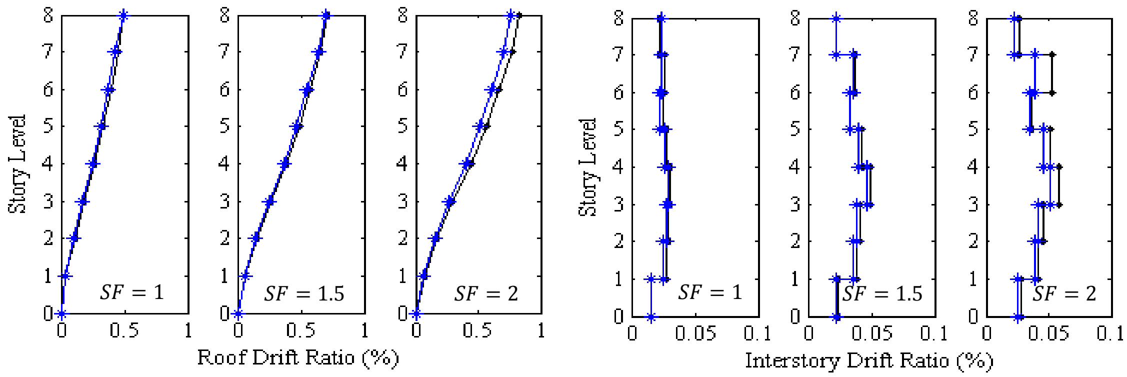


Figure 7. Predicted peak displacement and peak interstory drift demands by DSM compared to NL-RHA analyses

5 CONCLUSION

An improved direct procedure for seismic demands of MDOF bilinear system has been developed and its accuracy was verified by examples, and a response spectrum, called DDRS, has been developed and its applicability was tested for the selected example. The efficiency of the DSM method is evident; the designer needs only to have the DSM method for the design earthquake (s) to determine peak response of any structure, namely, base displacement and base shear. This method is applicable to a variety of uses such as a rapid evaluation technique for a large inventory of buildings, a design verification procedure for new construction, an evaluation procedure for an existing structure to identify damage states. The ductility demand is given by the direct estimation where the ductility calculated from the DDRS-η diagram matches the value associated with the period of the system. This method gives the deformation value consistent with the selected DDRS inelastic response spectrum, while retaining the attraction of graphical implementation of the ATC-40 methods.

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