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BUCKLING AND POST-BUCKLING BEHAVIOR OF BEAMS ON ELASTIC FOUNDATION MODELING BURIED PIPELINES

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ABSTRACT

Buckling and post-buckling behavior of beams resting on elastic foundation is addressed in the present study, as a first step towards modeling upheaval buckling of buried pipelines. The mathematical model used is that of a simply-supported Winkler beam supported laterally by uniformly distributed transverse springs, which is subjected to constant axial force over its length. Elastic critical buckling loads and corresponding eigenmodes are first obtained analytically, by formulating equilibrium equations in the deformed configuration and deriving and solving the corresponding buckling equation. The results are compared with results from linear buckling analyses of finite element models and indicate buckling mode cross-over with respect to soil stiffness. Then, geometrically nonlinear analyses with imperfections (GNIA) are performed, indicating descending post-buckling paths, thus unstable post-buckling behavior as well as buckling mode interaction for certain ranges of values of soil stiffness.

1 INTRODUCTION

Buried pipelines transporting oil products are structures of great financial, environmental and social importance. Such structures must adapt to eventual deformations of the surrounding soil, thus they may be severely damaged by large imposed permanent ground displacements triggered by landslides or seismic fault activation, causing combined axial and bending actions along the pipeline. Possible failure modes are tensile fracture at the welds between adjacent pipeline parts, local shell wall buckling in regions of high compressive stresses and upheaval buckling, which may be critical for relatively shallowly buried underground pipelines with low diameter to thickness ratio[1].

Upheaval buckling of buried pipelines was idealized by Yun and Kyriakides[2] as a long heavy beam on rigid foundation and formulae for bending moments and axial forces were extracted. Hobs[3] investigated the buckling of heated pipelines on rigid seabed and extracted analytical solutions for the critical buckling load and the corresponding buckling length. However, in reality soil is not rigid and its flexibility has to be taken into account to properly model upheaval buckling. Thus, buried pipelines prone to upheaval buckling should be modeled as beams resting on a deformable foundation. Experimental research has been conducted by several researchers to deal with pipeline upheaval buckling on elastic soil[4-5]. Wang et al.[6] adopted the model of a beam on elastic or plastic foundation to investigate thermal global buckling of buried pipelines.

The problem of beams supported on a deformable foundation is very common in engineering practice and its applications can be found in foundation engineering, buried structures etc. In Winkler's approach soil is modeled as a single layer and its behavior is approximated by a series of closely spaced, mutually independent, linear elastic transverse springs that provide resistance proportional to beam deflection. Timoshenko and Gere[7] showed for simply-supported beams resting on elastic foundation under concentrated axial compression load, that the critical buckling eigenmode changes with respect to soil stiffness, i.e. increasing soil stiffness leads to eigenmode cross-over. The buckling and post-buckling behavior of beams resting on an elastic foundation was analytically investigated by Kounadis et al.[8] who derived expressions of post-buckling equilibrium path for perfect 1-DOF models of such beams. Song and Li[9] dealt with thermal buckling and post-buckling of pinned-fixed beams on elastic foundation by introducing a so called "shooting method" to analytically solve the complex boundary condition problem. Additionally, the energy method was used to analytically describe post-buckling behavior with reference to buckling temperature.

The present study aims at addressing not only buckling loads, eigenmodes and eigenmode cross-over, but relate these with buckling and post-buckling response through nonlinear numerical analysis and extract useful remarks on pipeline buckling behavior with reference to elastic soil stiffness for a simply-supported beam, as the first step towards post-buckling behavior investigation of upheaval buckling of buried pipelines.

2 ANALYTICAL APPROACH - LINEAR BUCKLING ANALYSIS

2.1 Buckling loads and eigenmodes

Consider the simply-supported Euler-Bernoulli beam of length l and flexural rigidity EI, resting on Winkler foundation of stiffness k and axially compressed by constant load P, illustrated in Fig. 1. Denoting by $y(x)$ the transverse deflection of the beam, the governing fourth order differential equation of equilibrium is given by

$$EIy(x)^{''''} + Py(x)^{''} + ky(x) = 0 \tag{1}$$

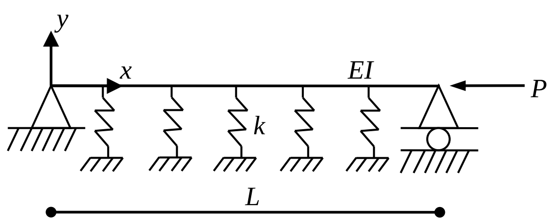

Figure 1. Simply-supported beam resting on elastic foundation under axial compression load

The general solution of the differential Eq. (1) is given by Eq. (2). Parameters A and B are denoted in Eq. (3) with $\alpha^2 = p/EI$ and $\beta^4 = k/4EI$.

$$y(x) = C_1 \cos Ax + C_2 \sin Ax + C_3 \cos Bx + C_4 \sin Bx \tag{2}$$

$$A = \sqrt{\left(\alpha^2 - \sqrt{\alpha^4 - 16\beta^4}\right)/2} \ , \ B = \sqrt{\left(\alpha^2 + \sqrt{\alpha^4 - 16\beta^4}\right)/2} \tag{3}$$

For the simply-supported beam the boundary conditions are

$$y(0) = 0, \ y(l) = 0, \ y^{''}(0) = 0, \ y^{''}(l) = 0 \tag{4}$$

The onset of buckling of the beam is determined by the solution of the linearized problem of Eq. (4) that yields equation

$$\left(A^2 - B^2\right) sinAl \cdot sinBl = 0 \tag{5}$$

The algebraic solution of Eq. (5) provides the critical buckling load of Eq. (6), where $n=1,2,...$ is the eigenmode number and P_E is the Euler critical buckling load for a simply-supported beam without elastic support.

$$P = \frac{n^2\pi^2 EI}{l^2} + \frac{kl^2}{n^2\pi^2} \text{ or } P = n^2 P_E + \frac{kl^2}{n^2\pi^2} \quad (6)$$

Substituting Eq. (6) to Eq. (2) the eigenmode equation of the simply-supported beam on elastic foundation is extracted and presented in Eq. (7). It should be noted that after algebraic manipulations the integration constant is erased from Eq. (6) given that the shape magnitude is unknown, only the eigenmode shape is of interest and term *sinAl≠0*. The first four eigenmode shapes are presented in Fig. 2, denoted according to symmetry about the center of the beam as 1S (1st symmetric), 1A (1st antisymmetric), 2S (2nd symmetric), 2A (2nd antisymmetric).

$$y(x) = \sin Bx - \frac{\sin Bl}{\sin Al}\sin Ax \quad (7)$$

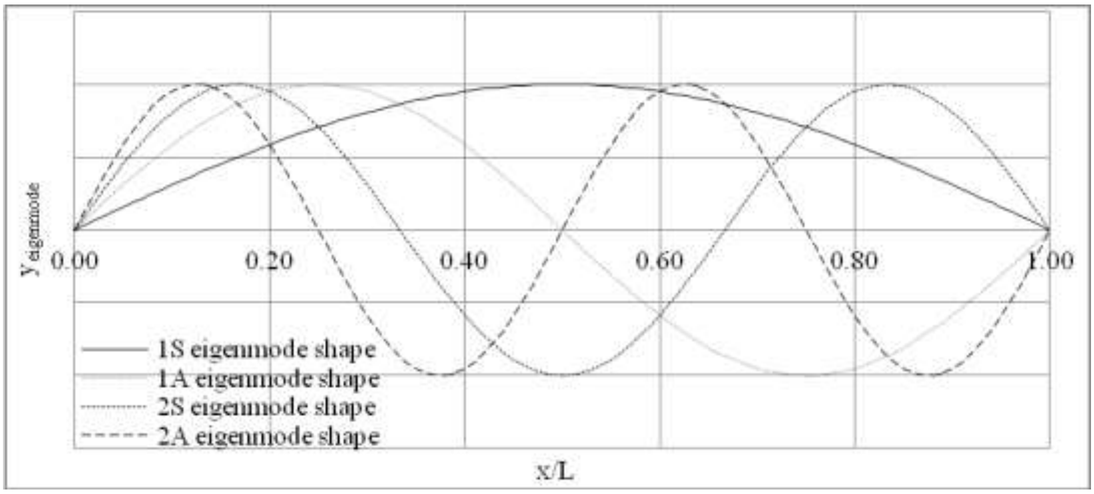

Figure 2. First four eigenmode shapes of simply-supported beam on elastic foundation

2.2 Eigenmode cross-over

Buckling behavior of beam resting on elastic foundation as outlined in section 2.1 is directly dependent on soil stiffness *k*, whose gradual increase leads to eigenmode cross-over. This is illustrated in Fig. 3 where soil stiffness is plotted on the horizontal axis, normalized with respect to beam length and flexural rigidity $\bar{k} = kl^4 / EI$ and elastic critical buckling load of the lower four eigenmodes is plotted on the vertical axis, normalized by Euler buckling load of a simply-supported beam without elastic support $\bar{P} = P / P_E$.

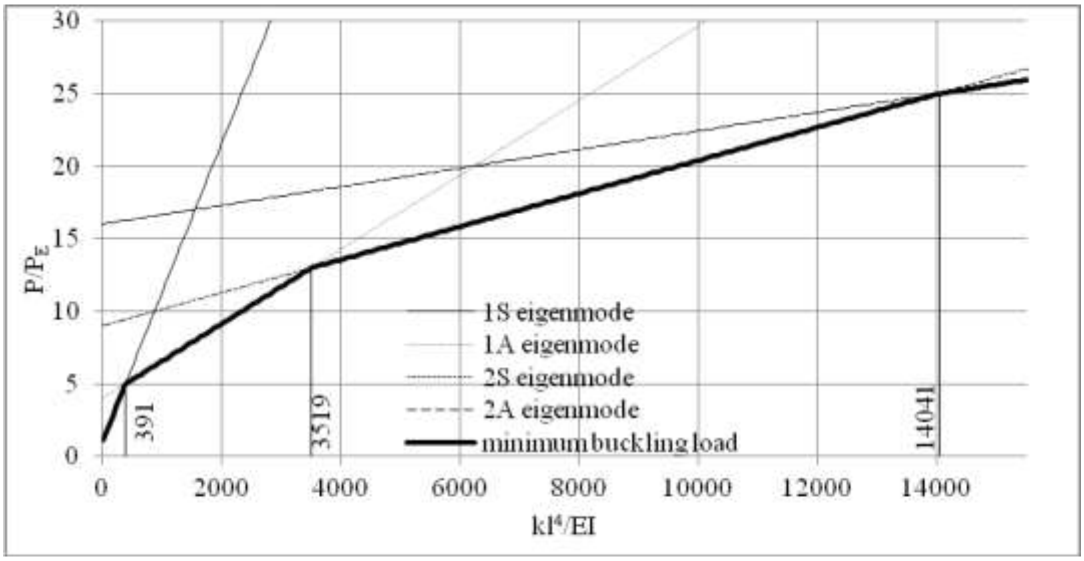

Figure 3. Elastic critical buckling load of the lower four eigenmodes vs. soil stiffness

Fig. 3 illustrates that increasing soil stiffness leads to proportional buckling load increase for all eigenmodes but with lower slope for higher modes, so that eigenmode cross-over takes place. Critical modes are 1S for $\bar{k}$ less than 391, 1A for $\bar{k}$ between 391 and 3519, 2S for $\bar{k}$ between 3519 and 14041, and 2A for $\bar{k}$ larger than 14041.

3 NUMERICAL APPROACH - GEOMETRICALLY NONLINEAR ANALYSIS

Numerical treatment of the problem is next carried out using commercial FEM software ADINA[10]. For this purpose a simply-supported beam is considered, featuring a typical cross-section CHS 33.7x2.0 and length L=5.00m. Beam material is elastic steel with Young's modulus E=210GPa and Poisson's ratio v=0.30m. Beam numerical simulation is carried out using Hermitian beam type finite elements with longitudinal mesh discretization equal to 0.05m, following a mesh density sensitivity analysis. Elastic foundation is modeled by transverse translational linear springs connecting beam and "ground" nodes, with the latter considered fixed. The beam is subjected to a compressive axial load applied at the roller edge, as in Fig. 1.

3.1 Linearized buckling analysis

At first, the model is used to obtain critical buckling loads and corresponding eigenmodes by means of Linearized Buckling Analysis (LBA). The results indicate an excellent match with those obtained analytically in the previous section.

3.2 Imperfection shapes for nonlinear analysis

It is well known that the presence of unavoidable imperfections may affect significantly the response of buckling-sensitive structures. In this work linear combinations of the first four eigenmodes are adopted as imperfection shapes and incorporated in geometrically nonlinear analyses (GNIA). The shape of eigenmodes is obtained by linearized buckling analyses presented in section 3.1. Linear combinations of eigenmode shapes are listed in Table 1 and are normalized so that their maximum equals *L/500*, which stands for a common engineering practice for steel members. The shape of imperfections is illustrated in Fig. 3 normalized with respect to beam length.

Table 1. Imperfections considered in GNIA

imperfection name	linear combination
I	1S+2S+1A+2A
II	1S-2S+1A+2A
III	1S+2S+1A-2A
IV	1S-2S+1A-2A

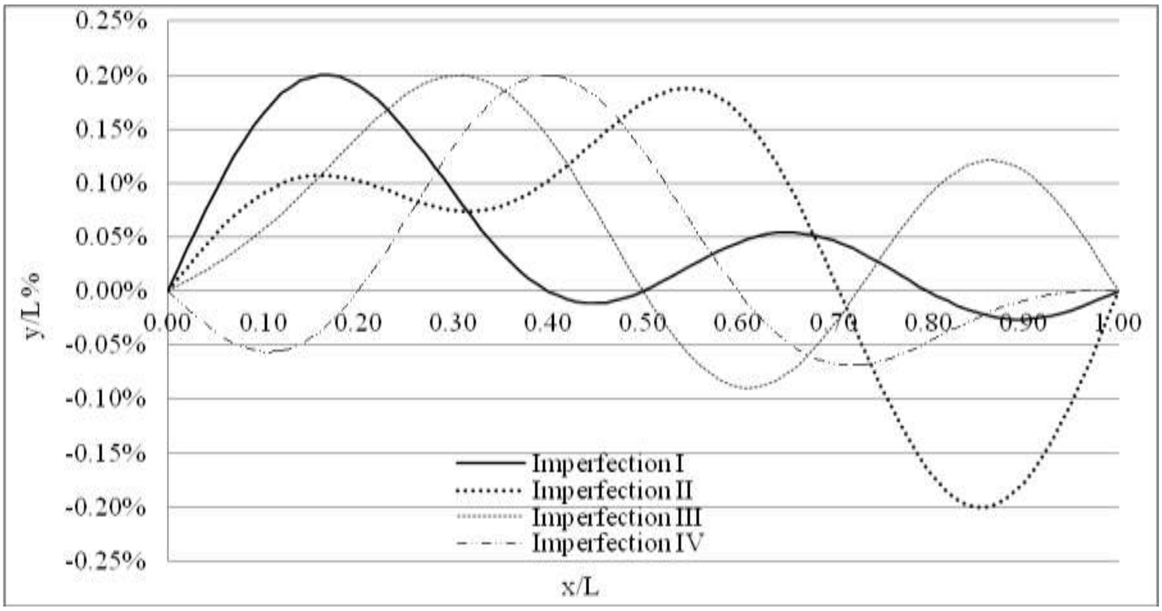

Figure 3. Imperfection shapes considered in GNIA

3.3 Geometrically Nonlinear Imperfection Analysis

In Geometrically Nonlinear Imperfection Analysis (GNIA) equilibrium equations are formulated in the deformed configuration of the structure that is allowed to differ significantly from the undeformed one. GNIA is very useful for investigating both buckling and particularly post-buckling behavior of the structure though the equilibrium path of a characteristic position on beam. For that purpose, the position with maximum transverse displacement (y_{max}) is selected. Four cases of soil stiffness, namely in case 1 eigenmode 1S is critical, case 2 refers to eigenmode cross-over from 1S to 1A, in case 3 eigenmode 1A is critical and case 4 refers to eigenmode cross-over from 1A to 2S. Every case is examined considering the four imperfection shapes defined in section 3.2. Linear combination of eigenmode shapes as imperfection shapes aims at quantifying the effects of imperfections in the structural response and detecting all possible imperfection sensitivities. In all cases the results are presented by means of equilibrium path, plotting on the horizontal axis the transverse displacement normalized with respect to beam length (y_{max}/L) and on the vertical axis the applied axial load normalized with respect to the linear critical buckling load (F/P_E) of the corresponding case. Moreover, the deformed shape of the beam at the end of the analysis is presented and compared to the shapes of initial imperfections and eigenmodes, leading to very interesting conclusions.

GNIA results are presented in Figs. 4 to 11. The first important observation is that equilibrium paths have descending post-buckling behavior in all cases that were investigated. Such unstable post-buckling behavior is crucial during design and should be taken into account through appropriate safety factors, as structure safety cannot rely on post-buckling strength. Moreover, as soil stiffness increases from case 1 to case 4 the difference between the linear buckling load and ultimate load from nonlinear analysis increases. Another common feature of all four soil stiffness cases is that the response is practically unaffected by the shape of initial imperfections.

Regarding the beam deformed shape at the end of the analysis, it is observed that it is not affected by imperfection shapes. In case 1, where the soil stiffness is such that eigenmode 1S is clearly critical the deformed shape at the end of the analysis is dominated by mode 1S, regardless of the shape of initial imperfections (Fig. 5). Similarly, in case 3, where the soil stiffness is such that eigenmode 1A is clearly critical the deformed shape at the end of the analysis is dominated by mode 1A (Fig. 9). On the contrary, in case 2, where the soil stiffness is such that cross-over between modes 1S and 1A takes place, the deformed shape at the end of the analysis is a mixture of modes 1S and 1A, also regardless of the shape of initial imperfections (Fig. 7). Similarly, in case 4, where the soil stiffness is such that cross-over between modes 1A and 2S takes place, the final deformed shape is a mixture of modes 1A and 2S, again regardless of the shape of initial imperfections (Fig. 11).

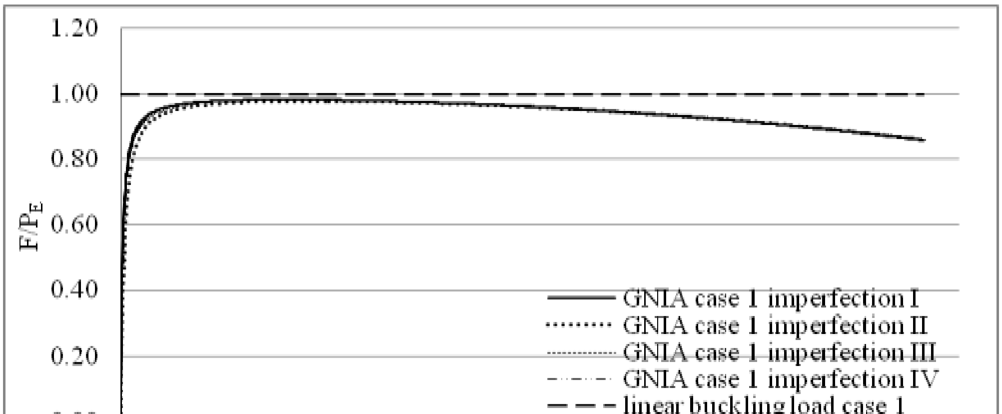

Figure 4. Case 1 - equilibrium path

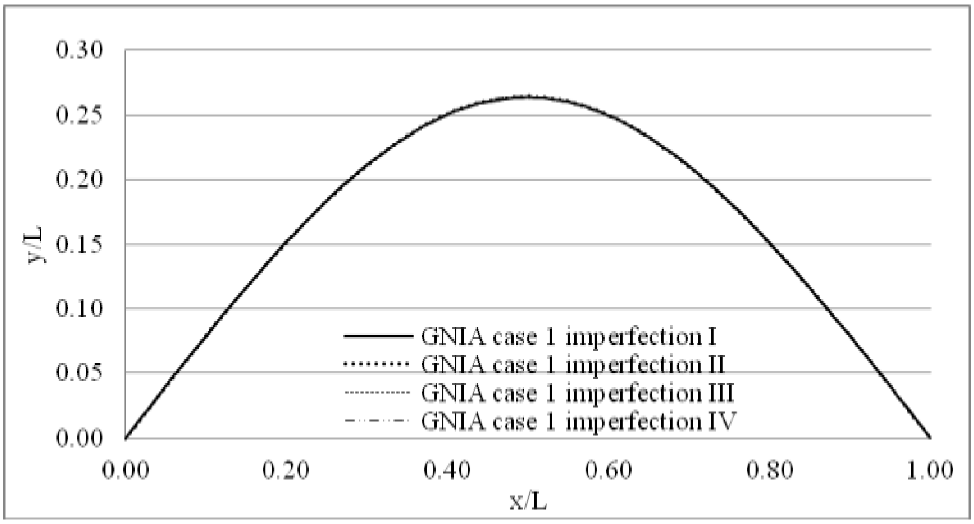

Figure 5. Case 1 - deformed shape

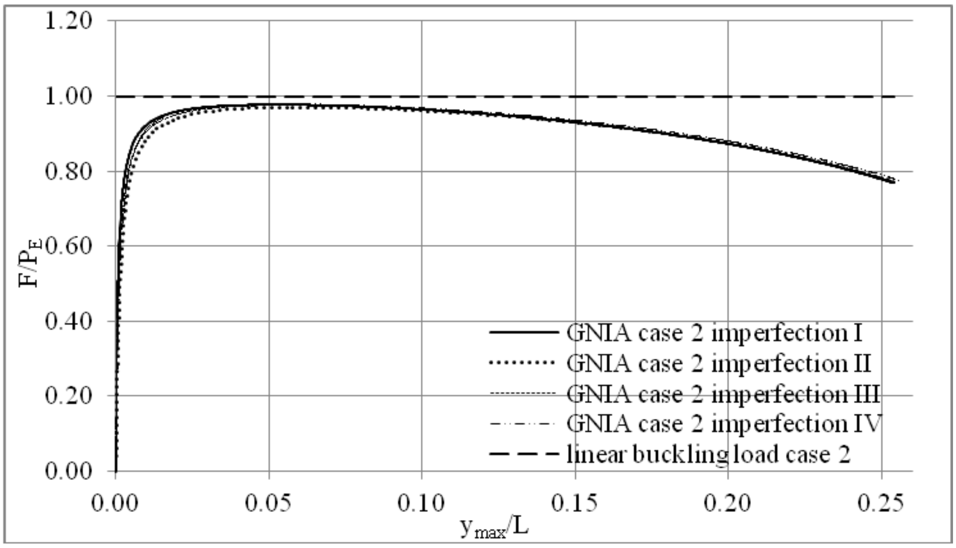

Figure 6. Case 2 - equilibrium path

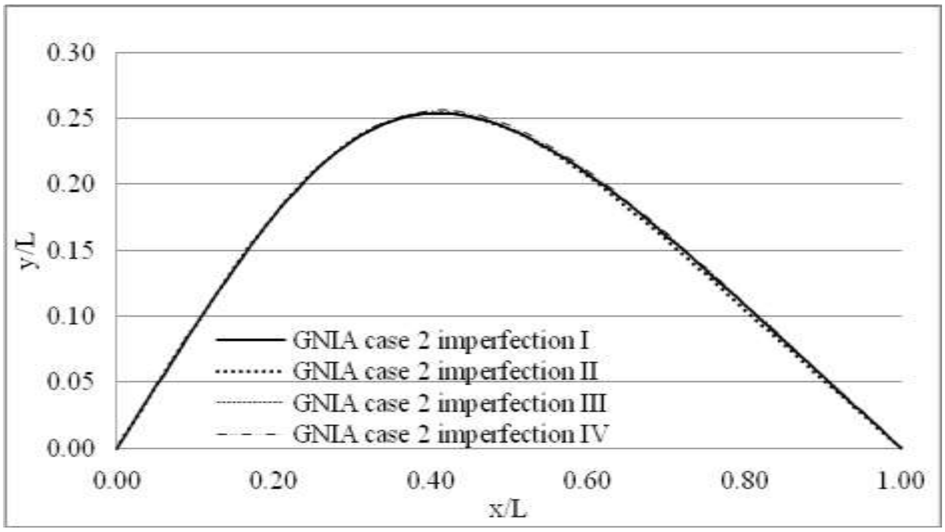

Figure 7. Case 2 - deformed shape

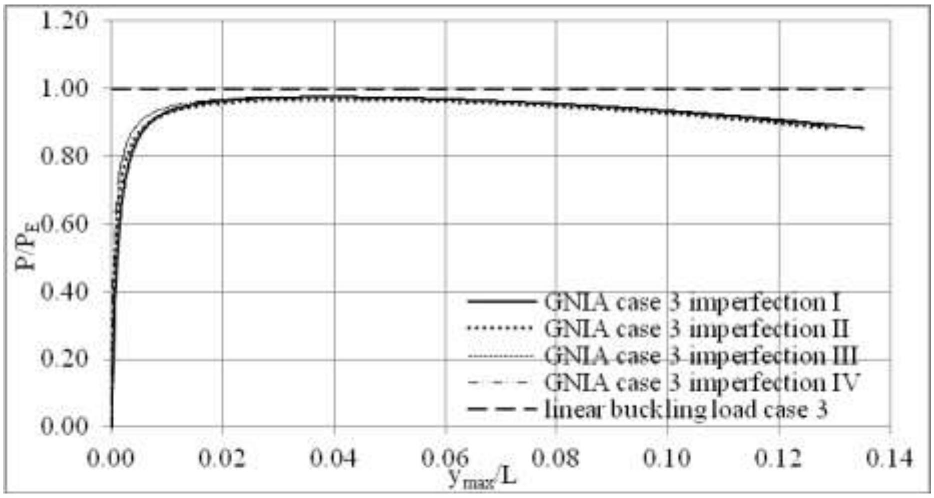

Figure 8. Case 3 - equilibrium path

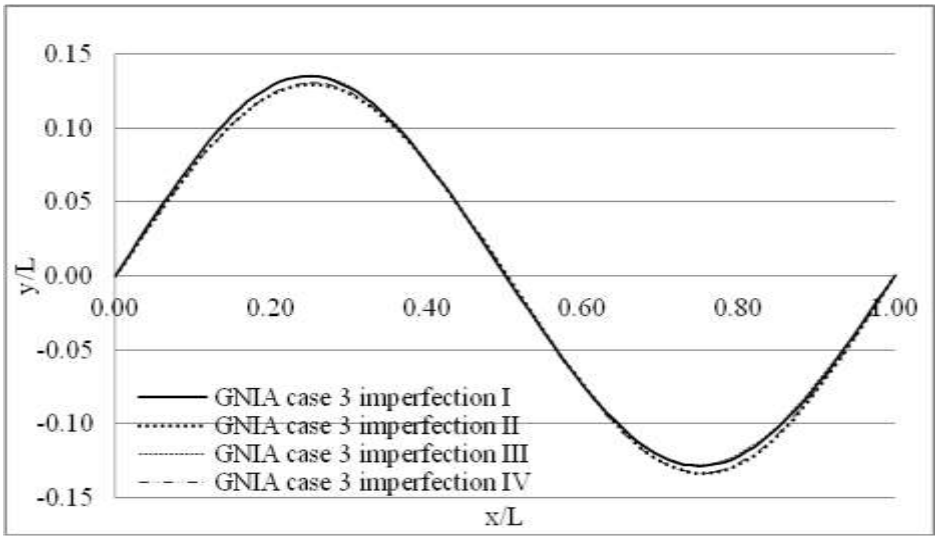

Figure 9. Case 3 - deformed shape

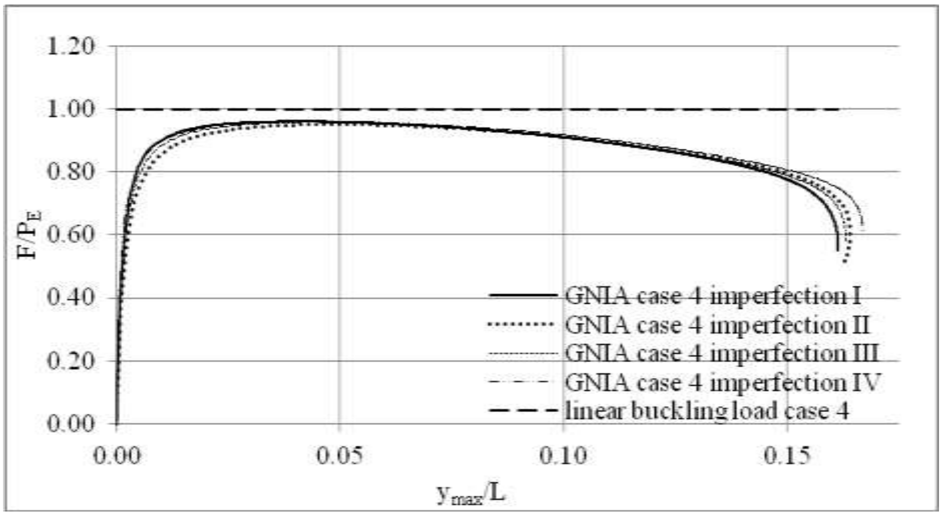

Figure 10. Case 4 - equilibrium path

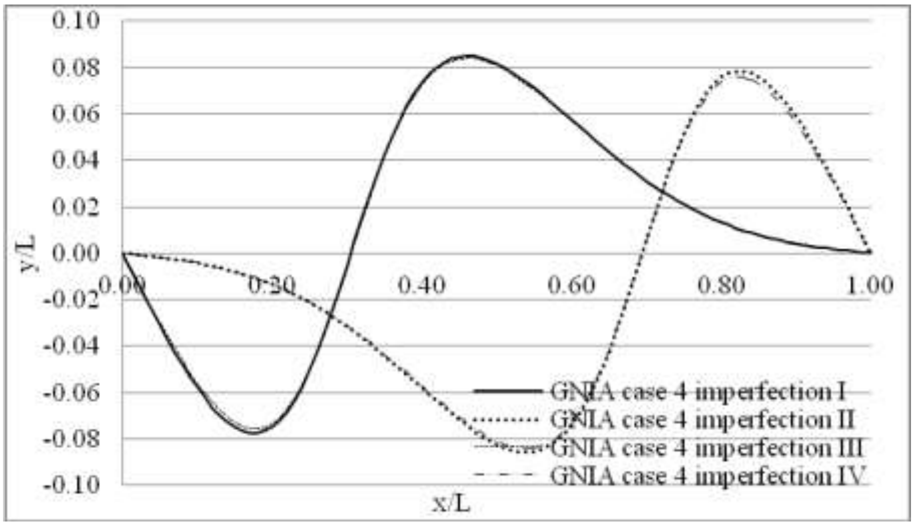

Figure 11. Case 4 - deformed shape

4 CONCLUSIONS

Flexural buckling of a simply-supported beam resting on elastic foundation is investigated both analytically and numerically, as a first step towards modeling upheaval buckling of buried pipelines. Transverse spring stiffness affects critical buckling loads and corresponding eigenmodes. Increase of soil stiffness leads to proportional increase of all critical buckling loads, but with lower proportionality ratios for higher modes, so that eigenmode cross-over takes place, which is examined analytically and numerically with linearized buckling analyses. Buckling and post-buckling behavior is then investigated numerically through geometrically nonlinear imperfection analysis. The descending equilibrium path in all considered cases proves the unstable post-buckling behavior of elastic beam resting on Winkler foundation. Moreover, higher soil stiffness is associated with increased difference between linearized buckling load and ultimate load. In cases where one buckling mode is clearly critical, the post-buckling beam deformed shape is dictated by the shape of the critical mode, while in cross-over cases it is a mixture of the shapes of crossing modes, regardless of the shape of imposed imperfections. The obtained remarks are useful in cases of buried pipelines relatively shallowly buried and prone to upheaval buckling due to axial compression. Future research in this area should take material nonlinearity of both soil and pipeline steel into account.

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