PHYSICAL MODELLING AND PARTICLE SWARM DESIGN OF COPLANAR WAVEGUIDE SQUARE SPIRAL INDUCTOR

N.I. Dib* and J.I. Ababneh*

Abstract

This paper presents simple lumped element equivalent circuit for the coplanar waveguide (CPW) square spiral inductor. The circuit is based on physical modelling which takes into consideration the parasitic effects inherent in the CPW spiral inductor. The obtained scattering parameters are in good agreement with full-wave and quasi-static results available in the literature. Such a simple circuit model is suitable for computer aided design (CAD) purposes, and could be used along with optimization techniques for synthesis purposes. Moreover, the particle swarm optimization (PSO) technique is used to design the CPW inductor. Specifically, given the desired inductance, the PSO is used to find the dimensions and the number of turns of the spiral.

Key Words

Particle swarm optimization, coplanar waveguide, inductor, circuit modelling

1. Introduction

With the growing demand for RF and mobile communications, many efforts concentrated on developing design rules for MMIC spiral inductors [1–20]. In general, the main parameter that has been studied is the inductance of the spiral. Most of these papers used the technique proposed by Greenhouse [10], which sums all the mutual and self inductances of the spiral segments using closed form expressions. Recently, three simple closed form expressions have been proposed in [18, 19] to evaluate the inductance of planar spirals of different shapes.

Most of the above-mentioned papers dealt with microstrip spiral inductors, while very few papers dealt with the coplanar waveguide (CPW) spiral inductors [14–17]. In [15], a rather complicated distributed element equivalent circuit was proposed to analyze the CPW spiral inductors. Though novel, the details of this technique are not fully presented in the paper. In [16], a lumped element equivalent circuit was proposed to model the CPW spiral inductor. The elements of this circuit were found using the quasi-static finite difference method (FDM). In [17], a square multilayered CPW spiral inductor was presented, measured, and analyzed using a full-wave package. However, no equivalent circuit model was proposed.

In this paper, simple lumped element equivalent circuit, similar to that used in [16], is used to model the CPW spiral inductor. The circuit is based on physical modelling which takes into consideration the parasitic effects inherent in the CPW spiral inductor. Closed form expressions are used to evaluate the elements in the equivalent circuit. The inductance expressions presented in [18, 19] are used to find the inductance of the CPW spiral inductor. Our results are in good agreement with the FDM results from [16], and full-wave results obtained using HFSS software [21].

Moreover, the particle swarm optimization (PSO) technique [22–27] is used to design the spiral inductor. Specifically, given the desired inductance, the PSO is used to find the dimensions and the number of turns of the spiral. The details of the PSO algorithm are presented in Section 3.

2. Equivalent Circuit Model

The CPW spiral inductor modelled here is shown in Fig. 1 (a three-turn spiral is shown). The spiral under consideration is a square one of side length \(d\), which is usually the case in practice because of the ease of its layout. An air-bridge is used to connect the inner port of the spiral to the output CPW. When analyzing this CPW square spiral inductor, the following parameters need to be specified:

1. \(d\): the outer dimension of the spiral.
2. \(N\): the number of turns (restricted to integer multiple of 0.5).
3. \(W\): the spiral strip width.
4. \(S\): the spacing between the spiral turns.
5. \(S_g\): the distance between the outermost turn and the ground plane.
6. \(H_a\): the height of the air-bridge.
7. \(W_a\): the width of the air-bridge.
8. \(t\): the metallization thickness.
9. \(\rho\): the metallization resistivity.

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The lumped element equivalent circuit shown in Fig. 2 is used to model the CPW spiral inductor. This circuit should be applicable up to the first resonant frequency of the inductor, which is the frequency range of interest as it represents the region where the spiral acts as an inductor. The lumped elements of this circuit are obtained as follows:

1. To evaluate the inductance $L$ of square spirals, any of the three simple expressions proposed in [18, 19] could be used. These expressions were fully tested in [18] and proved to be accurate enough for design and optimization of practical square spiral inductors. It should be noted that the value of the inductance $L$ is independent of the dielectric substrate, therefore, the formulas were obtained assuming spirals in free-space. For a grounded substrate, the expression presented in [11] can be used to evaluate the inductance $L$.

2. The resistance $R$ can be computed using the expressions from [1] or [13, 19]. These expressions take the skin effect into account. Thus, the resistance is a function of frequency.

3. The series capacitor $C_s$ models the parasitic capacitive coupling between the input and output ports of the inductor. Both the crosstalk between the adjacent turns and the overlap between the spiral and the air-bridge contribute to this capacitance $C_s$. The following approximate expression is used to evaluate this capacitance:

\[
C_s = C_{\text{coupling}} + C_{\text{air-bridge}}
\]  

where $L$ is the total mean length of the spiral, and $\ell_N$ is the mean length of the innermost turn. Essentially, $C_{\text{coupling}}$ is approximated as the side-wall parallel-plate capacitance between the turns [20]. The fringing effects are neglected. $\ell_N$ is subtracted from the total length $L$ since the innermost turn has no sidewall capacitance.

In addition, $C_{\text{air-bridge}}$ is approximated as a parallel-plate capacitance between the air-bridge and the spiral metallization. Such an approximation is valid since the height of the bridge is typically 3–5 $\mu$m which is very small compared to the bridge width and the spiral strip width. The factor $n$ is the number of turns that the air-bridge crosses ($n = N$ if $N$ is an integer, and $n = N - 0.5$ if $N$ is not an integer).

4. The parallel capacitors $C_1$ and $C_2$ are used to model the capacitance between the spiral (mainly the outermost turn) and the ground plane. Usually, the distance to the ground $S_g$ is large to reduce its effect on the spiral response. However, it has been shown in [15] that the ground effect needs to be included to get more accurate results. This is done by noticing that the path between the outermost turn and the ground resembles a slotline of width $S_g$ [15]. The capacitance of this slotline can be approximated as half the capacitance of a CPW ($C_{\text{CPW}}$) with centre conductor width $W$ and slot width $S_g$. After several numerical experiments and comparing the $S$-parameters obtained from our model to those available in the literature and those obtained using HFSS, the following expressions can be used to approximate $C_1$ and $C_2$:

\[
C_1 = \left( \frac{C_{\text{CPW}}}{2} \right) \left( \frac{4d}{4} \right)
\]

\[
C_2 = \frac{C_1}{\sqrt{N+1}}
\]

It should be noted that the values of these parallel capacitances have to be different so that the phases of $S_{11}$ and $S_{22}$ are not equal.

3. PSO Algorithm

PSO is a stochastic optimization technique that was first developed in 1995 by Eberhart and Kennedy [22–27]. It is a global optimization algorithm that has been effectively used to solve multidimensional discontinuous optimization problems in a variety of fields [28]. The algorithm is an attempt to simulate the swarming behaviour of birds, bees, fish, etc. It aims at increasing the probability of encountering the global minimum (or maximum), without performing an exhaustive search of the entire parameter space.
space. The performance of the PSO is comparable to other stochastic optimization technique like genetic algorithm (GA) and simulated annealing [27]. In addition, the PSO outperformed the GA in certain instances [29]. One of the key advantages of PSO is the ease of implementation in both the context of coding and parameter selection. The algorithm is much simpler and intuitive to implement than complex, probability-based selection and mutation operators required for evolutionary algorithms such as the GA.

Similar to evolutionary algorithms, the PSO starts with an initial population of individuals (to be termed swarm of particles). Each individual (particle) in the swarm is randomly assigned an initial position and velocity. The position of the particle is an N-dimensional vector that represents a possible set of the unknown parameters to be optimized. Each particle in the swarm starts from its initial position at its initial velocity with the goal of finding the position with global minimum (or maximum). During the algorithm search, the velocity and position of each particle is updated based on the individual and the swarm experience. The goodness of the new particle position (possible solution) is measured by evaluating a suitable objective or fitness function. This process of updating velocities and positions continues and results in that one of the particles in the swarm finds the position with best fitness (global minimum or maximum). Eventually, all the particles will be drawn to this position since they will not be able to find a better one.

The main steps of the PSO algorithm can be summarized in the following steps:

1. **Definition of the solution space**: The minimum and maximum values for each dimension in the $N$-dimensional optimization problem are specified to define the solution space of the problem. The solution space is the accepted parameters values and it is usually governed by the typical values of the parameters and their constraints.

2. **Definition of the fitness function**: The fitness function is a problem-dependent parameter which represents a method that can be used to evaluate and measure the goodness of a position ($N$-dimensional vector) that represents a valid solution of the problem. It should be carefully selected to represent the goodness of the solution and return a single number.

3. **Random initialization of the swarm positions and velocities**: The positions and velocities of the particles are randomly initialized. However, it is preferred that they are randomly initialized within the solution space for faster convergence. The number of particles is problem dependent, and for most engineering problems a swarm size of 30 is good enough [30]. To complete the initialization step, the initial position of each particle is labelled as the particle’s best position (pbest). In addition, the position of the particle with best fitness among the whole swarm is labelled as the global best position (gbest). In this context, the word best could mean highest or lowest depending on whether the problem at hand is a minimization or maximization problem.

4. **Update of velocity and position**: The following substeps are carried out for each particle:

   (a) **Velocity update**: The velocity of the $m$th particle in the $n$th dimension ($v_{mn}^t$) is updated according to the following equation:

   $$v_{mn}^{t+1} = w v_{mn}^{t} + c_1 R_1 (pbest_{mn}^{t} - x_{mn}^{t}) + c_2 R_2 (gbest_{mn}^{t} - x_{mn}^{t})$$

   where $x_{mn}$ represents the position of the $m$th particle in the $n$th dimension. The superscripts $t$ and $t−1$ denote the time index of the current and the previous iterations, $R_1$ and $R_2$ are two random numbers between 0 and 1 generated by uniformly distributed random functions. The relative weights of the personal best position versus the global best position are specified by the parameters $c_1$ and $c_2$, respectively. Both $c_1$ and $c_2$ are typically set to a value of 2.0 [31]. The parameter $w$ is called the “inertial weight”, and it is a number in the range [0–1] that specifies the weight by which the particle’s current velocity depends on its previous velocity, and the distance between the particle’s position and its personal best and global best positions. It has been shown in [30] that the PSO algorithm converges faster if $w$ is linearly damped with iterations, for example starting at 0.9 at the first iteration and finishing at 0.4 in the last iteration. Guidelines for selecting and optimizing the PSO parameters are detailed in [31–36].

   (b) **Position update**: The position of the $m$th particle in the $n$th dimension is updated according to:

   $$x_{mn}^{t+1} = x_{mn}^{t} + \Delta t v_{mn}^{t}$$

   where $\Delta t$ represents a given time step (usually chosen to be one).

   (c) **Fitness evaluation**: The fitness of the updated $N$-dimension position in the previous step is evaluated. If the returning number is better than that of the pbest, this updated $N$-dimension position is labelled as the new pbest. In addition, if the returning number is better than that corresponding to gbest, the updated $N$-dimension position is also labelled as the new gbest.

5. **Checking the termination criterion**: The algorithm may be terminated if the number of iterations reaches a pre-specified maximum number of iterations or the returning number corresponding to gbest is close enough to a desired number. If none of these conditions is satisfied, return to step 4.

In this work, the PSO is used to synthesize the spiral inductor. The goal is to find the spiral inductor parameters ($N$, $d$, $W$, $S$) to achieve a desired inductance value $L_{des}$. The fitness function for the problem at hand is formulated as:

$$Fitness = |L_{des} - L_{cal}|$$
Figure 3. Magnitude and phase of $S_{21}$ for the CPW spiral inductor obtained using the circuit model (solid lines) compared to results from [16] (symbols). $W = 25 \mu m$, $S = 5 \mu m$, $H_a = 3 \mu m$, $W_a = 25 \mu m$, $S_g = 50 \mu m$, $t = 3 \mu m$, $h = 635 \mu m$, $\epsilon_r = 13$. $d = 185 \mu m$ for $N = 2.5$; $d = 240 \mu m$ for $N = 3.5$; $d = 300 \mu m$ for $N = 4.5$.

where $L_{cal}$ is the calculated inductance using the three approximate expressions given in [18]. In our implementation of the PSO algorithm, a swarm size of 25 particles and number of iterations between 500 and 1000 were used. Both parameters $c_1$ and $c_2$ were set to a value of 2. The inertia weight was initially set to 0.9, and decreased linearly to 0.4 during the iteration process. This allows the initial large value to enhance the global exploration, while the final low value facilitates local exploration of the solution space. The results of this implementation of the PSO to design the spiral inductor are given in Section 5.

4. Circuit Model Results

First, the CPW spiral inductors analyzed in [16] are modelled. Fig. 3 shows the scattering parameters obtained from the circuit model compared to those obtained using FDM [16]. It can be seen that the agreement is very good which validates our proposed circuit model. It should be noted that the CPW spirals analyzed in [16] are not exactly square. The inductance values for the analyzed spirals are approximately as follows: $L = 0.8$ nH for the $N = 2.5$ spiral, $L = 1.7$ nH for the $N = 3.5$ spiral, and $L = 3.3$ nH for the $N = 4.5$ spiral.

As another example, Fig. 4 shows the scattering parameters for a 1.94 nH CPW spiral inductor with the following dimensions: $N = 3$, $d = 300 \mu m$, $S = 10 \mu m$, $W = 30 \mu m$, $S_g = 70 \mu m$, $H_a = 3 \mu m$, $W_a = W$, and $t = 0 \mu m$. Results obtained using HFSS [21] are also shown in the same figure. Our results are in good agreement with the HFSS results. It can be seen that the phase of $S_{11}$ is different from the phase of $S_{22}$ since the spiral is a non-symmetric structure. Other spirals with the same dimensions but with different number of turns were also analyzed and a similar agreement was obtained.

5. PSO Results

The PSO algorithm was used to design the spiral inductor by finding some (or all) of its main parameters ($N$, $d$, $W$, $S$) that result in a certain desired inductance value $L_{des}$. Two different cases were considered: In the first case, some of the spiral inductor parameters were set to some fixed values (desired or manufacture settings) and the remaining parameters were obtained using the PSO. In the other case, all the spiral inductor main parameters are searched by the PSO to obtain a desired inductance value. In both cases, the search process is constrained to some acceptable typical values ($2 \leq N \leq 15$, $100 \mu m \leq d \leq 480 \mu m$, $2 \mu m \leq W \leq 0.3d$, $2 \mu m \leq S \leq 3W$) as in [18]. A wide range of inductors (0.5–100 nH), can be designed using these typical values. Tables 1–4 give the results of designing several spiral inductors. In each table, the values of the remain-
Table 1
Design of Spiral Inductor Using the PSO
\((N = 5, \ d = 200 \ \mu m, \ W = 5 \ \mu m)\)

<table>
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<td>8</td>
<td>1.9</td>
<td>8.014</td>
<td>1.9</td>
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</table>

\(S\) in \(\mu m\), and inductance in nH.

Table 2
Design of Spiral Inductor Using the PSO
\((N = 10, \ d = 300 \ \mu m)\)

<table>
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Dimensions \(W\), and \(S\) in \(\mu m\), and inductance in nH. Two sets of valid solutions are shown for each case.

Table 3
Design of Spiral Inductor with \(N = 8\) Using the PSO

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Dimensions \(d\), \(W\), and \(S\) in \(\mu m\), and inductance in nH. Two sets of valid solutions are shown for each case.

Table 4
Complete Design of Spiral Inductor Using the PSO

<table>
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Dimensions \(d\), \(W\), and \(S\) in \(\mu m\), and inductance in nH. Two sets of valid solutions are shown for each case.
6. Conclusions

Simple lumped element equivalent circuit for the CPW square spiral inductor have been proposed and studied. This circuit takes into consideration the parasitic effects inherent in the CPW spiral inductor. The obtained results are in good agreement with those published in the literature, and HFSS results. Such a simple model is appropriate for CAD purposes. Moreover, it has also been demonstrated that the design of spiral inductors can be easily performed using the PSO technique.

References


[21] HFSS Software, Ansoft Corporation, PA, USA.


Biographies

*Nihad I. Dib* obtained his B.Sc. and M.Sc. degrees in EE from Kuwait University in 1985 and 1987, respectively. He obtained his Ph.D. degree in EE (major in Electromagnetics) in 1992 from University of Michigan, Ann Arbor. Then, he worked as an assistant research scientist in the radiation laboratory at the same school. In September 1995, he joined the EE Department at Jordan University of Science and Technology (JUST) as an assistant professor, and became an associate professor in September 2000. From July 2001 to March 2003, he was a senior research engineer at Ansoft Corporation, NJ, USA, on sabbatical leave from JUST. His research interests are mainly in computational electromagnetics and modeling of passive microwave circuits. Dr. Dib is a senior member of the IEEE.

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