Electromagnetics II

Waveguides

Prepared By:

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Chapter 12
Waveguides

- Studied TEM waves:
  1. $E_x = H_z = 0$
  2. $E \perp H$ & both $\perp \hat{z}$
  3. $u_p = c = \frac{c}{\sqrt{\varepsilon_r}}$ (lossless)
  4. no cutoff frequency

- TEM waves are not the only mode of guided waves. There are TE & TM waves

  TE: $H_z \neq 0 \& E_x = 0$
  TM: $H_z = 0 \& E_x \neq 0$

Both have cutoff frequencies $f_c$. Below $f_c$, waves (modes) are not excited.

- Waveguides:
  - only one conductor
  - can not support TEM
  - will use EM theory to study them
  - less attenuation compared to T.L.'s

- Dielectric waveguides:
  - no conductor
  - e.g. optical fibers
assumed that \( \sigma_d = 0 \) and \( \varepsilon_d = \infty \).

In the source-free dielectric region:

\[
\nabla^2 \vec{E} + k^2 \vec{E} = 0, \quad k = \omega \sqrt{\varepsilon_e} \\
\nabla^2 \vec{H} + k^2 \vec{H} = 0
\]

Each equation is a set of three eq's, e.g. \( E_x, E_y, E_z \).

For \( E_z \):

\[
\nabla^2 E_z + k^2 E_z = 0 \\
\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0
\]

Let \( E_z(x,y,z) = X(x) Y(y) Z(z) \)

Substitute & divide by \( XYZ \)

\[
\Rightarrow \frac{1}{X} \frac{d^2X}{dx^2} + \frac{1}{Y} \frac{d^2Y}{dy^2} + \frac{1}{Z} \frac{d^2Z}{dz^2} + k^2 = 0
\]

\( \text{fn. of x} \quad \text{fn. of y} \quad \text{fn. of z} \)
each term should be a constant.

\[
\frac{1}{X} \frac{d^2 X}{dx^2} = -k_x^2 \Rightarrow \frac{d^2 X}{dx^2} + k_x^2 X = 0
\]

\[
\frac{1}{Y} \frac{d^2 Y}{dy^2} = -k_y^2 \Rightarrow \frac{d^2 Y}{dy^2} + k_y^2 Y = 0
\]

\[
\frac{1}{Z} \frac{d^2 Z}{dz^2} = \gamma^2 \Rightarrow \frac{d^2 Z}{dz^2} - \gamma^2 Z = 0
\]

\[
- k_x^2 - k_y^2 + \gamma^2 = -k^2 = - \omega^2 \epsilon 
\]

Solution:

\[
X(x) = C_1 \cos(k_x x) + C_2 \sin(k_x x)
\]

\[
Y(y) = C_3 \cos(k_y y) + C_4 \sin(k_y y)
\]

\[
Z(z) = C_5 \frac{z}{\epsilon}^2 + C_6 \frac{z}{\epsilon}^{2\gamma}
\]

\[
E_2(x, y, z) = X \cdot Y \cdot Z
\]

- assume propagation in +z direction

\[
\Rightarrow C_5 = 0
\]

\[
Z(z) = C_6 \frac{z}{\epsilon}^{2\gamma}
\]

\[
E_2 = (A_1 \cos(k_x x + A_2 \sin(k_x x))(A_3 \cos(k_y y + A_4 \sin(k_y y)) \frac{z}{\epsilon}^{2\gamma}
\]

- Similarly,

\[
H_2 = (B_1 \cos(k_x x + B_2 \sin(k_x x))(B_3 \cos(k_y y + B_4 \sin(k_y y)) \frac{z}{\epsilon}^{2\gamma}
\]

- can solve also for other field components.
derive them from $E_2 \& H_2$, as follows:

\[ \mathbf{\nabla} \times \mathbf{E} = -j \omega \mu \mathbf{H} \quad \text{\&} \quad \mathbf{\nabla} \times \mathbf{H} = j \omega \epsilon \mathbf{E} \]

\[
\begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\sigma \\
E_x & E_y & E_z \\
\end{vmatrix} = -j \omega \mu \mathbf{H} \\
\]

\[
\Rightarrow \left\{
\begin{align*}
\frac{\partial E_z}{\partial y} + \gamma E_y &= -j \omega \mu H_x \quad (1) \\
-\gamma E_x - \frac{\partial E_z}{\partial x} &= -j \omega \mu H_y \quad (2) \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j \omega \mu H_z \quad (3)
\end{align*}
\right.

\[
\Rightarrow \left\{
\begin{align*}
\frac{\partial H_x}{\partial y} + \gamma H_y &= j \omega \epsilon E_x \quad (4) \\
-\gamma H_x - \frac{\partial H_z}{\partial x} &= j \omega \epsilon E_y \quad (5) \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j \omega \epsilon E_z \quad (6)
\end{align*}
\right.
\]

Can express all components in terms of $E_2 \& H_2$:

\[
\text{e.g. from (1) \& (5)}
\]

\[
\Rightarrow H_x = \frac{1}{\hbar^2} \left[ j \omega \epsilon \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x} \right]
\]

\[
\gamma \epsilon E_y = -\frac{1}{\hbar^2} \left[ \gamma \frac{\partial E_z}{\partial y} - j \omega \mu \frac{\partial H_z}{\partial x} \right]
\]

\[
\text{from (2) \& (4)}
\]

\[
\Rightarrow H_y = -\frac{1}{\hbar^2} \left[ \gamma \frac{\partial H_z}{\partial y} + j \omega \epsilon \frac{\partial E_z}{\partial x} \right]
\]

\[
\gamma \epsilon E_x = -\frac{1}{\hbar^2} \left[ \gamma \frac{\partial E_z}{\partial x} + j \omega \mu \frac{\partial H_z}{\partial y} \right]
\]

where \( \hbar^2 = \gamma^2 + k^2 = \gamma^2 + \omega^2 \mu \epsilon = k_x^2 + k_y^2 \)
Types of waves (or modes):

1. TEM mode: \( E_z = H_z = 0 \)

   Previous eq's \( \Rightarrow \) all components = zero

   Unless

   \( h^2 = 0 \)

   \( \Rightarrow \gamma^2 + k^2 = 0 \)

   \( \gamma = jk = j\omega\sqrt{\mu\varepsilon} \quad \text{as expected} \).

   There must be at least 2 conductors to support TEM.

   Waveguides can not support TEM.

2. TE mode: \( E_z = 0 \) & \( H_z \neq 0 \)

3. TM mode: \( E_z \neq 0 \) & \( H_z = 0 \)

4. Hybrid mode: \( E_z \neq 0 \) & \( H_z \neq 0 \) (e.g. in dielectric guides)

12.3 TM modes

B.C.'s: \( E_z (x=0, y) = 0 \Rightarrow A_1 = 0 \)

\( E_z (x=a, y) = 0 \Rightarrow k_x = \frac{m\pi}{a} \), \( m=1,2,\ldots \)

\( E_z (x, y=0) = 0 \Rightarrow A_3 = 0 \)

\( E_z (x, y=b) = 0 \Rightarrow k_y = \frac{n\pi}{b} \), \( n=1,2,\ldots \)

\( \therefore E_z = E_0 \sin \left( \frac{m\pi}{a} x \right) \sin \left( \frac{n\pi}{b} y \right) \quad \text{as expected} \)
Can find other components using previously derived eq's with $H_z = 0$.

\[ E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} \]

\[ E_x = -\frac{\gamma}{h^2} \left( \frac{mx}{a} \right) E_0 \cos\left( \frac{mx}{a} x \right) \sin\left( \frac{nx}{b} y \right) e^{-\gamma z} \]

(see book, p. 548 for other components)

\[ h^2 = k_x^2 + k_y^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 = \gamma^2 + \omega^2 \mu \varepsilon \]

- Each set of $m$ and $n$ gives a different field pattern, referred to as $TM_{mn}$ mode.

- Neither $m$ nor $n$ can be zero, since if any of them is zero $E_z = 0 \Rightarrow$ all components are zeros.

$TM_{11}$ is the lowest order mode of all $TM_{mn}$ modes.

$m$ denotes the number of half-cycles variations in $x$-direction

$n$ denotes the number of half-cycles variations in $y$-direction
Propagation constant $\gamma$:

$$\gamma = \sqrt{\left(\frac{m_x}{a}\right)^2 + \left(\frac{n_x}{b}\right)^2 - k^2}, \quad k^2 = \omega^2 \mu \epsilon$$

(1) **cutoff**

If $$k^2 = \left(\frac{m_x}{a}\right)^2 + \left(\frac{n_x}{b}\right)^2 = \omega^2 \mu \epsilon = \frac{4\pi^2 f^2 \mu \epsilon}{\lambda}$$

$$\Rightarrow \gamma = 0 \Rightarrow \alpha = \beta = 0$$

The frequency that causes this is called the cutoff frequency $f_c$:

$$f_c = \frac{1}{2\pi \sqrt{\mu \epsilon}} \sqrt{\left(\frac{m_x}{a}\right)^2 + \left(\frac{n_x}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi \sqrt{\mu \epsilon}} \frac{h}{2\pi \lambda}$$

So, each $TM_{mn}$ mode has its own $f_c$.

(2) If $$k^2 < \left(\frac{m_x}{a}\right)^2 + \left(\frac{n_x}{b}\right)^2 \quad \text{(i.e.} \; f < f_c)$$

$$\Rightarrow \gamma = \alpha \quad \& \quad \beta = \text{zero}$$

No wave propagation, called **evanescent modes**.

(3) If $$k^2 > \left(\frac{m_x}{a}\right)^2 + \left(\frac{n_x}{b}\right)^2 \quad \text{(i.e.} \; f > f_c)$$

$$\Rightarrow \gamma = j \beta \quad \& \quad \alpha = \text{zero}$$

There is a propagating wave.
\[ \beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \]

- \( f_c \) is the frequency below which there is no wave propagation. The frequency has to be larger than \( f_c \) to have a propagating mode.

\[
(f_c)_{mn} = \frac{1}{2\pi \sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}
\]

\[
(f_c)_{m_n} = \frac{u}{2} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}
\]

where \( u = \frac{1}{\sqrt{\mu\varepsilon}} \)

- Notice that \( TM_{11} \) mode has lowest \( f_c \) of all \( TM \) modes.

\[
(f_c)_{11} = \frac{u}{2} \sqrt{(\frac{1}{a})^2 + (\frac{1}{b})^2}
\]

- if \( f < (f_c)_{11} \) \( \Rightarrow \) no mode exists
- if \( f > (f_c)_{11} \) \( \Rightarrow \) \( TM_{11} \) is alive.

\[ \beta = \sqrt{\omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \sqrt{\omega^2 \mu \varepsilon - \omega_c^2 \mu \varepsilon} \]

\[ = \omega \sqrt{\mu \varepsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \]

\[ \beta = k \sqrt{1 - (f_c/f)^2} \]

- Each mode has its own \( \beta \).
Define cutoff wavelength $\lambda_c$ as

$$\lambda_c = \frac{u}{f_0} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Guide wavelength $\lambda_g$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{u}{k}$$

Note $\lambda_g$ is larger than $\lambda$, always.

Wavelength in the guide wavelength in unbounded medium

$$\lambda_g^2 = \frac{\lambda^2 f^2}{f^2 - f_c^2} = \frac{u^2}{f^2 - f_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{f^2}{u^2} - \frac{f_c^2}{u^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\therefore \frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

Define phase velocity $u_p$:

$$u_p = \frac{\omega}{\beta} = \frac{u}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow u_p = \frac{\lambda_g}{\lambda} u$$

Note $u_p > u$. 
Define wave impedance

\[ \gamma_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega e} \]

\[ \Rightarrow \gamma_{TM} = \frac{k}{\omega e} \sqrt{1 - (\frac{f_c}{f})^2} = \gamma \sqrt{1 - (\frac{f_c}{f})^2} \]

where \( \gamma = \sqrt{\frac{\mu}{\varepsilon}} \).

Summary: For \((TM)_{mn}\) modes:

- Cutoff freq. \( (f_c)_{mn} = \frac{u}{a} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \), \( u = \frac{1}{\sqrt{\mu e}} = \frac{c}{\sqrt{\varepsilon_r \varepsilon}} \)

- Phase const. \( \beta = k \sqrt{1 - (\frac{f_c}{f})^2} \), \( k = \omega \sqrt{\mu e} = \frac{w}{c} \sqrt{\varepsilon_r \varepsilon} \)

- Cutoff wavelength \( (\lambda_c)_{mn} = \frac{u}{(f_c)_{mn}} \)

- Guide wavelength \( \lambda_g = \frac{2\pi}{\beta} \)

- Phase velocity \( u_p = \frac{\omega}{\beta} \)

- Wave impedance \( \gamma_{TM} = \frac{\beta}{\omega e} \)
12.4 TE modes: \( E_x = 0 \) \& \( H_z \neq 0 \)

\[
H_z = \left( B_1 \cos k_x x + B_2 \sin k_x x \right) \left( B_3 \cos k_y y + B_4 \sin k_y y \right) e^{-\gamma z}
\]

B.C.'s:

\[
E_y(x=0, y) = 0 \Rightarrow \left. \frac{\partial H_z}{\partial x} \right|_{x=0} = 0 \Rightarrow B_2 = 0
\]

\[
E_y(x=a, y) = 0 \Rightarrow \left. \frac{\partial H_z}{\partial x} \right|_{x=a} = 0 \Rightarrow k_x = \frac{m_x}{a}
\]

\[
E_x(x, y=0) = 0 \Rightarrow \left. \frac{\partial H_z}{\partial y} \right|_{y=0} = 0 \Rightarrow B_4 = 0
\]

\[
E_x(x, y=b) = 0 \Rightarrow \left. \frac{\partial H_z}{\partial y} \right|_{y=b} = 0 \Rightarrow k_y = \frac{n_x}{b}
\]

\[
\therefore H_z = H_0 \cos \left( \frac{m_x}{a} x \right) \cos \left( \frac{n_y}{b} y \right) e^{-\gamma z}
\]

- Can derive other components from \( H_z \) (see book, p. 553).

- \( k_x = \frac{m_x}{a} \), \( k_y = \frac{n_x}{b} \) ← same as those for TM modes

  ⇒ the expressions in p. 10 for

  \( f_c, \beta, \lambda_c, \alpha_y \) & up still hold here.

except \( \eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{we}{\beta} \)

- For TE modes: either \( m \) or \( n \) can be zero, but not both.

- If \( m=n=0 \) ⇒ all field components = zero!!
(f_c)_{mn} = \frac{u}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}

the mode with lowest f_c could be TE_{10} or TE_{01} depending on "a" & "b".

(f_c)_{10} = \frac{u}{2a} \quad (f_c)_{01} = \frac{u}{2b}

As a standard practice, a > b

⇒ TE_{10} is the mode with the lowest f_c among ALL modes.

⇒ TE_{10} is the dominant mode.

TE_{10} is of practical importance.

(f_c)_{TE_{10}} = \frac{u}{2a} \quad (\lambda_c)_{TE_{10}} = \frac{u}{f_c} = 2a

TE_{10}:

E_x = \text{zero since } E_x \sim \sin \left(\frac{n\pi}{b} y\right)

E_y = -j \frac{u\mu a}{\pi} H_0 \sin \left(\frac{\pi}{a} x\right)

H_x = j\beta a \quad H_0 \sin \left(\frac{\pi}{a} x\right)

H_y = \text{zero since } H_y \sim \sin \left(\frac{nx}{b} y\right)

H_z = H_0 \cos \left(\frac{\pi}{a} x\right)

E_z = \text{zero (TE mode)}
Note: \( E \) has only one component

\[ f_c = \frac{u}{a} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \]

- \( f_{c_{10}} = \frac{3E8}{2a} = 6.55 \text{ GHz} \)
- \( f_{c_{01}} = \frac{u}{2b} = 14.7 \text{ GHz} \)
- \( f_{c_{11}} = \frac{u}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 16.1 \text{ GHz} \)
- \( f_{c_{20}} = \frac{u}{2a} \sqrt{\left(\frac{2}{a}\right)^2} = \frac{u}{a} = 13.1 \text{ GHz} \)

**Note:** In X-band, only the \( \text{TE}_{10} \) mode is excited (dominant mode).

**Note:** Here \( (f_c)_{20} < (f_c)_{01} \) ! Not always true!!

If \( a < 2b \) \( (f_c)_{20} > (f_c)_{01} \). 

Ex. WG-16 waveguide, X-band waveguide (8-12 GHz)

\( a = 2.29 \text{ cm}, \ b = 1.02 \text{ cm}, \text{ air-filled} \)
Ex. 12.2

Air filled rectangular waveguide,

\[ 5 \times 2 \text{ cm} \quad (i.e. \quad a = 5 \text{ cm} \quad b = 2 \text{ cm}) \]

\[ f = 15 \text{ GHz} \quad \beta = j \beta \]

\[ E_z = 20 \times \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{5 \pi y}{b} \right) \quad e^{-j \beta z} \]

(a) What mode is it??

Clearly, it is a \( TM_{mn} \) mode

\[ k_x = \frac{m \pi}{a} = 40 \pi \quad \Rightarrow \quad m = \frac{40 a}{2} \]
\[ k_y = \frac{n \pi}{b} = 50 \pi \quad \Rightarrow \quad n = \frac{50 b}{1} \]

\[ \therefore \quad TM_{21} \quad \text{mode} \]

(b) Find \( \beta \).

\[ \beta = \omega \sqrt{\mu \varepsilon} \quad \frac{1}{\sqrt{1 - \left( \frac{f_c}{f} \right)^2}} \]

\[ \left( f_c \right)_{TM_{21}} = \frac{u}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} \quad \text{with} \quad m = 2 \quad \& \quad n = 1 \]
\[ \quad \& \quad u = 3 \times 10^8 \]

\[ \Rightarrow \quad \left( f_c \right)_{21} = 9.605 \quad \text{GHz} \]

\[ \therefore \quad \beta = 241.3 \quad \text{rad/\mu} \]

(c) Find \( \frac{E_x}{H_y} \)

\[ \frac{E_x}{H_y} = \eta_{TM} = \frac{\beta}{\omega \varepsilon_0} = 289.2 \quad (\Omega) \]
It is shown that the TE\textsubscript{10} mode can be represented as the sum of two plane waves propagating along zigzag paths between the walls \( x = 0 \) and \( x = a \).

\[
\begin{array}{c}
\rightarrow \\
\downarrow \\
\downarrow \\
\rightarrow \\
\end{array}
\]

- Group velocity (energy velocity) is defined as:

\[
\frac{u_g}{u} = \frac{1}{\frac{\partial \beta}{\partial \omega}} = u \sqrt{1 - \left(\frac{f_c}{f}\right)^2} < u
\]

- \( u_p u_g = u^2 \)
\[ \alpha = \alpha_c + \alpha_d \]

\[ \alpha_d = \frac{\sigma_d^2 \gamma}{2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad \text{holds for any waveguide mode} \]

\[ \alpha_c \bigg|_{TE_{10}} = \frac{2 R_s}{b \gamma \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \left( \frac{1}{2} + \frac{b}{a} \left(\frac{f_c}{f}\right)^2 \right) \]

\[ R_s = \frac{1}{\alpha_c} S \]

\[ \alpha_c \to \infty \quad \text{as} \quad f \to f_c \]

\[ a = 22.86 \text{ mm} \quad \Rightarrow \quad f_c \bigg|_{TE_{10}} = 6.55 \text{ GHz} \quad \text{copper walls} \]

\[ f_c \bigg|_{TE_{20}} = 13.1 \text{ GHz} \]

For \( b = 10.2 \text{ mm} \), \( \alpha_c \bigg|_{\text{minimum}} \) occurs around 14 GHz.
12.7 Mode excitation:

Probe excitation for TE_{10} mode

12.8 Waveguide Resonators

- Resonators are used for energy storage.
- Used in klystrons, bandpass filters, & wave meters.
- RLC circuit elements are inefficient at high frequencies.
- Waveguide resonators have very high Q.

Closed from all sides
\( E_z = E_0 \sin \left( \frac{mx}{a} \right) \sin \left( \frac{ny}{b} \right) \cos \left( \frac{pz}{c} \right) \)

Note that \( E_z (x=0) = E_z (x=a) = \text{zero} \quad \text{for B.C.'s} \)
\( E_z (y=0) = E_z (y=b) = \text{zero} \)

However, \( E_z (z=0) \neq \text{zero} \)
\( \& \quad E_z (z=c) \neq \text{zero} \)

\[ k^2 = k_x^2 + k_y^2 + k_z^2 \quad \Rightarrow \quad \omega^2 \mu \varepsilon = \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2 \]

\[ f_r = \frac{\mu}{\omega} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2} \]

Resonant frequency of the TM\(_{mnp}\) mode.

Neither \( m \) nor \( n \) can be zero. But, \( p \) can be zero.

\( \text{TM}_{110} \) is the mode with lowest \( f_r \) among TM modes.

TE modes

\( H_z = H_0 \cos \left( \frac{mx}{a} \right) \cos \left( \frac{ny}{b} \right) \sin \left( \frac{pz}{c} \right) \)

Same equation for \( f_r \) holds here too.

Either \( m \) or \( n \) (but not both) can be zero.
\( p \) cannot be zero.
- The mode with lowest \( f_r \) among **ALL** modes depends on \( a, b, \) & \( c \) values. It is called the dominant mode.
- The dominant mode is either \( TM_{\text{110}} \) or \( TE_{\text{101}} \) or \( TE_{\text{011}} \).

- If \( a > b > c \)

\[
(f)_mnp = \frac{n}{2} \sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2 + (\frac{p}{c})^2}
\]

Clearly \( TM_{\text{110}} \) is the dominant mode.

- If \( a > c > b \)

\[\Rightarrow TE_{\text{101}} \text{ is the dominant.}\]

- If \( a < b < c \)

\[\Rightarrow TE_{\text{011}} \text{ is the dominant.}\]

- If \( a = b = c \) \[\Rightarrow TM_{\text{110}}, TE_{\text{101}} \& TE_{\text{011}} \text{ have same } f_r \text{.} \]
  (called degenerate modes).

**Suggested problems**

12. 4, 5, 6, 8, 10, 11, 14, 17, 21, 23, 32, 34