1. (20 points) If the electric flux density \( \vec{D} \) is given as

\[
\vec{D} = \begin{cases} 
  r \hat{\hat{r}} & r \leq 1 \text{ m} \\
  \frac{1}{r} \hat{\hat{r}} & r > 1 \text{ m}
\end{cases}
\]

(a) (5 points) Find volume charge density \( \rho \) at \( r = 0.5 \text{ m} \) and \( r = 1.5 \text{ m} \).

(b) (5 points) Calculate the total electric flux leaving a unit length cylinder of radius 0.5 m.

(c) (5 points) Find the total electric energy stored in a unit length cylinder of radius 2 m. (Assume \( \varepsilon_0 \) everywhere)

(d) (5 points) What infinite length line charge \( \rho_l \) should be placed along the z-axis to cause \( \vec{D} \) to be zero for \( r > 1 \text{ m} \).

2. (10 points) An infinite current sheet \( \vec{J}_x = 5 \hat{\hat{x}} \) (A/m) coinciding with the \( xy \)-plane separates air \( (z > 0) \) from a medium with \( \varepsilon_r = \mu_r = 2 \) \((z < 0)\).

Given that

\[
\vec{H}_{air} = \hat{\hat{x}} \ 30 + \hat{\hat{y}} \ 40 + \hat{\hat{z}} \ 20 \quad (A/m).
\]

Find \( \vec{H} \) just below the \( xy \)-plane.

3. (10 points) Let \( \vec{B}(x) = 0.5 \hat{\hat{z}} \) as shown in the figure.

The position of the sliding bar is given by the equation:

\[
x(t) = 6 t^2 + t \quad (m)
\]

where \( t \) is time in seconds.

If the separation between the rails is 10 cm, find the voltmeter reading when the bar is located at \( x = 1 \text{ m} \).
1. **(20 points)** A surface current density \( \mathbf{J}_s = J_0 \sin(\theta) \hat{\phi} \) (A/m) is flowing on the surface of a conducting sphere of radius \( b \).

(a) **(5 points)** Find the magnetic vector potential \( \mathbf{A} \) at the center of the sphere.

(b) **(15 points)** Find the magnetic field density \( \mathbf{B} \) at the center of the sphere.

5. **(20 points)** In general, given a force \( \mathbf{F}(\mathbf{r}) \) distributed over an object and given a particular reference point \( \mathbf{r}_0 \) (see figure), the net torque about \( \mathbf{r}_0 \) is

\[
\mathbf{T} = \int (\mathbf{r} - \mathbf{r}_0) \times d\mathbf{F}(\mathbf{r})
\]

(a) **(5 points)** Show that the torque will be independent of \( \mathbf{r}_0 \) when the net, or total force, acting on an object is zero.

Now, assume that a surface current density

\[
\mathbf{J}_s(\mathbf{r}) = \begin{cases} 
\frac{1}{r} \hat{\phi} & a \leq r \leq b \\
0 & \text{elsewhere}
\end{cases}
\]

flows on the surface of a disk in the xy-plane. The disk is located in a uniform magnetic field \( \mathbf{B} = B_0 \hat{z} \) (T).

(b) **(5 points)** Show that the net force acting on the disk is zero.

(c) **(10 points)** Using the above expression for the torque, calculate the net torque acting on the disk around the origin (i.e., \( \mathbf{r}_0 = 0 \)). Note that the torque is independent of the reference point since the net force is zero.
To make it more challenging, only final answers are given:

1. (a) 2 \( \text{μC/m}^3 \) & zero
   (b) 0.5 \( \pi \) \( \text{μC} \)
   (c) 0.3346 \( J \)
   (d) -2 \( \pi \) \( \text{μC/m} \)

2. \( \mathbf{H} = \hat{x} 30 + \hat{y} 45 + \hat{z} 10 \) \( \text{A/m} \)

3. -0.25 \( \text{V} \)

4. \( \mathbf{A} = \text{zero} \), \( \mathbf{B} = \hat{z} \frac{2}{3} \mu_0 J_0 \) \( \text{T} \)

5. (a) see notes
   (b) \( \mathbf{F} = \text{zero} \)
   (c) \( \mathbf{T} = -\hat{z} B_0 \pi (b^2 - a^2) \) \( \text{N.m} \)