Applications of Static Fields

1. Introduction

Now that we have discussed the fundamentals of electrostatic and magnetostatic fields, we can explain some of the applications of static fields. It may appear unusual that we have devoted a complete chapter to discussing the applications of static fields when some of these applications could easily have been included in the preceding chapters. We have several reasons for doing so:

1. To discuss some of the applications in their entirety requires knowledge of both electrostatic and magnetostatic fields. For instance, the acceleration of a charged particle in a cyclotron is accomplished by an electric field, whereas the rotation is imparted by a magnetic field.

2. By presenting the major applications of static fields in one chapter we hope to convince the reader of their importance. We have seen some recently published textbooks that tend to skip over the subject of static fields as if they are of no significance.

3. If there is not enough time to discuss the applications of static fields in the classroom, we presume that this chapter epitomizes a very good reading assignment for the student.

2. Deflection of a Charged Particle

One of the most common applications of electrostatic fields is the deflection of a charged particle such as an electron or a proton in order to control its trajectory. Devices such as the cathode-ray oscilloscope, cyclotron, ink-jet printer, and velocity selector are based on this principle. Whereas the charge of an electron beam in a cathode-ray oscilloscope is constant, the charge on the fine particles of ink in an ink-jet printer varies with the character to be printed. In any case, the deflection of a charged particle is accomplished by maintaining a potential difference between a pair of parallel plates.

Consider a charged particle with charge $q$ and mass $m$ moving in the $x$ direction with a velocity $u_x$, as shown in Figure 6.1. At time $t = 0$ the charged particle enters the region between the pair of parallel plates held at a potential difference of $V_0$. Ignoring
the effects of fringing of the electric field lines, the electric field intensity within the parallel plates is

$$\vec{E} = -\frac{V_0}{L} \hat{a}_z$$

where $L$ is the separation between the two parallel plates. The force acting on the charged particle due to the electric field is

$$\vec{F} = q \vec{E}$$

which is in the downward direction. Neglecting the effect of gravitational force on the charged particle, the acceleration in the $z$ direction is

$$a_z = \frac{qV_0}{mL}$$

(6.1)

Thus, the velocity of the charged particle within the parallel plates in the $z$ direction is

$$u_z = a_z t$$

(6.2)

because $u_z = 0$ at time $t = 0$.

The displacement of the charged particle in the $z$ direction is

$$z = \frac{1}{2} a_z t^2$$

(6.3)

because $z = 0$ at time $t = 0$. However, the displacement of the charged particle in the $x$ direction in time $t$ is

$$x = u_x t$$

(6.4)

The time taken by the charged particle to exit the region between the parallel plates is

$$T = \frac{d}{u_x}$$

(6.5)

Thus, the trajectory of the charged particle within the parallel plates, from (6.3) and (6.4), is

$$z = -\frac{qV_0}{2mL} \left[ \frac{x}{u_x} \right]^2$$

(6.6)

which is an equation for a parabola.

**Example 6.1**

A potential difference of 1.5 kV is maintained between two parallel plates that are held 10 cm apart. An electron with a kinetic energy of 2 keV enters the deflection plates at right angles to the electric field. If the plates are 20 cm long, determine (a) the time taken by the electron to exit the plates and (b) the deflection of the electron as it exits the plates.
**Solution**

From the kinetic energy of the electron, we can determine the velocity of the electron at time $t = 0$ in the $x$ direction as

$$\frac{1}{2}mv^2 = 2 \times 10^3 \times 1.6 \times 10^{-19}$$

Substituting $m = 9.11 \times 10^{-31}$ kg for the electron, we obtain

$$v = 26.52 \times 10^6 \text{ m/s}$$

a) The time taken by the electron to exit the parallel plates is

$$T = \frac{20 \times 10^{-2}}{26.52 \times 10^6} = 7.54 \times 10^{-9} \text{ s} \quad \text{or} \quad 7.54 \text{ ns}$$

b) The deflection of the electron, from (6.6), is

$$z = \frac{1.6 \times 10^{-19} \times 1.5 \times 10^3}{2 \times 9.1 \times 10^{-31} \times 0.1} \left( \frac{20 \times 10^{-2}}{26.52 \times 10^6} \right)^2$$

$$= 74.97 \times 10^{-3} \text{ m} \quad \text{or} \quad 74.97 \text{ mm}$$

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**EXERCISES**

6.1 If the separation between the plates in Example 6.1 is reduced to 5 cm, what must be the applied voltage to cause a deflection of 0.5 cm? Sketch the velocity vs. time and velocity vs. distance curves for the electron during its travel between the parallel plates.

6.2 How long must the plates be in Example 6.1 so that the deflection is 2 cm? Compute the time now taken by the electron to exit the parallel plates. What is its velocity in the direction of its deflection at the time of exit?

6.3 Cathode-Ray Oscilloscope

The essential features of a cathode-ray oscilloscope are illustrated in Figure 6.2. The tube itself is of glass and is highly evacuated. The cathode emits electrons when it is heated by a heating filament. These electrons are then accelerated toward an anode that is held at a potential of several hundred volts with respect to the cathode. The anode has a small hole that allows a narrow beam of electrons to pass through it. The accelerated electrons then enter a region where they can be deflected in both the horizontal and vertical directions in a similar manner as discussed in Section 6.2. Finally, the electron beam bombards the inner surface of a screen coated with a substance (phosphor) that emits visible light.

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**Figure 6.2**

Basic elements of a cathode-ray oscilloscope
Let us assume that the initial velocity of the electron as it is being emitted from the cathode’s surface is zero. If \( V_0 \) is the potential difference between the anode and the cathode, the velocity of the electron as it exits the anode can be obtained from the gain in its kinetic energy as

\[
\frac{1}{2}mV_1^2 = eV_1
\]

or

\[
u_1 = \left(\frac{2e}{m} V_1\right)^{1/2}
\]  
(6.7)

Let us assume that there exists no potential difference between the horizontal deflection plates and that the top vertical deflection plate is held at a potential of \( V_0 \) with respect to the lower plate. The electron passes undisturbed through the horizontal deflection plates and experiences a force in the positive \( z \) direction in the region between the vertical deflection plates. The vertical displacement as the electron exits the vertical deflection region \( x = d \), as shown in Figure 6.3, from (6.6), is

\[
z_1 = \frac{eV_0}{2md} \left[ \frac{d}{u_x} \right]^2
\]  
(6.8)

The corresponding velocity in the \( z \) direction for \( x = d \) is

\[
u_z = \frac{eV_0}{mdu_x}
\]  
(6.9)

whereas the velocity in the \( x \) direction remains unchanged. As the electron exits the vertical deflection region, it moves in a straight-line path as \( u_x \) and \( u_z \) are both constant. The velocity \( \mathbf{u} \) now makes an angle \( \theta \) with the \( x \) axis, where

\[
\tan \theta = \frac{u_z}{u_x}
\]  
(6.10)

The time required by the electron to travel a distance \( D \) on emerging from the deflection plates to the screen is

\[
\tau_2 = \frac{D}{u_x}
\]

Thus,

\[
z_2 = u_z \tau_2 = \frac{eV_0}{md} \left[ \frac{1}{u_x} \right]^2
\]  
(6.11)
Therefore, after substituting for \( u_x \) in this equation, the total vertical displacement of the electron as it strikes the screen is

\[
z = z_1 + z_2 = \frac{d}{2L} \left[ 0.5d + D \right] \frac{V_0^2}{V_1}
\] (6.12)

It is evident from (6.12) that if the potential difference between the anode and the cathode is held constant, the deflection of the electron is proportional to the potential difference between the vertical deflection plates. By applying a potential difference between the horizontal deflection plates, we can cause the electron to move in the \( y \) direction. Therefore, the point at which the electron beam will strike the screen depends upon the vertical and the horizontal deflecting voltages.

**EXERCISES**

6.3 Two parallel plates are held 5 cm apart. An electron is released at the surface of the negatively charged plate and strikes the surface of the opposite plate in 12.5 ns. Find (a) the velocity of the electron at the instant it strikes the positively charged plate, (b) the acceleration of the electron, (c) the potential of the positively charged plate with respect to the negatively charged plate, and (d) the electric field intensity within the plates.
6.4 Ink-Jet Printer

A novel printing technique based upon the electrostatic deflection principle has been developed to increase the speed of the printing process and enhance the print quality. The resulting printer is called an ink-jet printer. In an ink-jet printer, a nozzle vibrating at ultrasonic frequency sprays ink in the form of very fine, uniformly sized droplets separated by a certain spacing. These droplets acquire charge proportional to the character to be printed while passing through a set of charged plates, as depicted in Figure 6.4. With a fixed potential difference between the vertical deflection plates, the vertical displacement of an ink droplet is proportional to its charge. A blank space between characters is achieved by having no charge imparted to the ink droplets (in this case, the ink droplets are collected by the ink reservoir). In a cathode-ray oscilloscope the horizontal deflection of the electron is obtained by constantly changing the potential difference between the horizontal deflection plates. However, in an ink-jet printer the printer head is moved horizontally at a constant speed, and the characters can be formed at the rate of 100 characters per second (cps).

As there are very few moving parts, ink-jet printers are very quiet and reliable in operation compared with impact printers. Also, impact printers limit printing to only those characters that are on the print-wheel, whereas any character can be formed with ink-jet printers, making them very versatile. As you may have guessed, the equations that determine the trajectory of an ink droplet are exactly the same as those for an electron in a cathode-ray oscilloscope.

**Example 6.3**

An ink droplet of diameter 0.02 mm attains a charge of \(-0.2\) pC as it passes through the charging plates at a speed of 25 m/s. The potential difference between the vertical deflection plates held 2 mm apart is 2 kV. If the length of each deflection plate is 2 mm, and the distance from the exit end of the deflection plate to the paper is 8 mm, determine the vertical displacement of the ink droplet. Assume that the density of the ink is 2 grams per cubic centimeter.

**Solution**

The mass of the ink droplet is

\[
m = \frac{4\pi}{3} \left( \frac{1}{2} \times 0.02 \times 10^{-3} \right)^3 \times 2 \times 10^{-3} = 8.38 \times 10^{-12} \text{ kg}
\]
The total vertical deflection is
\[
z = \frac{qd}{mL} \left[ \frac{1}{V_0} \right] [0.5d + D] \\
= \frac{2 \times 10^{-13} \times 2 \times 10^{-3} \times 2000}{8.38 \times 10^{-12} \times 2 \times 10^{-5}} \left[ \frac{1}{25^2} \right] [0.5 \times 2 + 8] \times 10^{-3} \\
= 0.69 \text{ mm}
\]

**EXERCISES**

6.5 Compute the velocity of the ink droplet in Example 6.3 as it strikes the paper. What is the total time taken by the ink droplet from the instant it enters the deflection plate region and strikes the paper?

6.6 An ink droplet of diameter 0.01 mm attains a charge of \(-2\) pC as it passes through the charging plates at a speed of 20 m/s. The potential difference between the vertical deflection plates held 5 mm apart is 200 V. If the length of each deflection plate is 1.5 mm, and the distance from the exit end of the deflection plate to the paper is 12 mm, determine the vertical displacement of the ink droplet. Assume that the density of the ink is 2 grams per cubic centimeter.

### 6.5 Sorting of Minerals

The principle of electrostatic deflection is also employed by the mining industry to sort oppositely charged minerals. For example, in an ore separator, phosphate ore containing granules of phosphate rock and quartz is dropped onto a vibrating feeder, as illustrated in Figure 6.5. The vibrations cause the granules of phosphate rock to rub against the particles of quartz. During the rubbing process each quartz granule acquires a positive charge and each phosphate particle acquires a negative charge. The sorting of the oppositely charged particles is accomplished by passing them through an electric field set up by a parallel-plate capacitor.

![Figure 6.5](image)

Ore separator

To develop an expression for the trajectory of the charged particle within the parallel-plate capacitor region, let us assume that the mass and the charge of the quartz particle
are $m$ and $q$, respectively. Let the initial velocity of each particle be zero at the instant it enters the charged region between the parallel plates, as shown in Figure 6.6. Then $u_x = 0$ and $u_z = 0$ at $t = 0$. The force of gravity will impart acceleration in the $x$ direction. At any time $t$, the velocity and the distance travelled in the $x$ direction are

$$u_x = \frac{dx}{dt} = gt$$  \hspace{1cm} (6.13)

and

$$x = \frac{1}{2} gt^2$$  \hspace{1cm} (6.14)

The motion of the charged particle in the $z$ direction can be described as

$$u_z = \frac{d}{dt} V_0$$  \hspace{1cm} (6.15)

and

$$u_z = at$$  \hspace{1cm} (6.16)

From (6.14) and (6.17), we obtain the trajectory of each charged particle as

$$z = \frac{1}{2} at^2$$  \hspace{1cm} (6.17)

From (6.14) and (6.18), we obtain the trajectory of each charged particle as

$$z = \frac{1}{2} at^2$$  \hspace{1cm} (6.18)

This equation reveals that the trajectory of a charged particle is a straight line within the parallel-plate region. The time taken by the charged particle to exit the parallel-plate region is

$$T = \left[ \frac{2d}{g} \right]^{1/2}$$  \hspace{1cm} (6.19)

For any time $t \geq T$, the velocity of the charged particle in the $z$ direction is constant and

$$u_z = \frac{dV_0}{mL} \left[ \frac{2d}{g} \right]^{1/2} \quad \text{for} \quad t \geq T$$  \hspace{1cm} (6.20)

and

$$z = u_z t \quad \text{for} \quad t \geq T$$  \hspace{1cm} (6.21)

From (6.14) and (6.21), we can express $z$ in terms of $x$ as

$$z^2 = \frac{2}{g} u_z^2 x \quad \text{for} \quad t \geq T$$  \hspace{1cm} (6.22)

which is an equation for a parabola. Thus, a charged particle follows a straight-line path within the parallel plates and a parabolic path thereafter.

**EXAMPLE 6.4**

A quartz particle with a mass of 2 grams acquires a charge of 100 nC on the vibrating feeder. The particle then falls freely at the middle of the top edge of the parallel plates, which are held at a potential difference of 10 kV. If the plates are 2 m in length and are 50 cm apart, determine the position and the velocity of the particle at the end of the plates.

**Solution**

The time taken by the quartz particle to leave the plates, from (6.19), is

$$T = \left[ \frac{2 \times 2}{9.81} \right]^{1/2} = 638.55 \text{ ms}$$
From (6.15), the acceleration of the quartz particle within the parallel-plate region is

\[ a_z = \frac{100 \times 10^{-9} \times 10 \times 10^3}{2 \times 10^{-3} \times 0.5} = 1.0 \text{ m/s}^2 \]

The distance travelled in the \( z \) direction in time \( t = T \) is

\[ z = \frac{1}{2} \times 1.0 \times (638.55 \times 10^{-3})^2 = 0.204 \text{ m or 20.4 cm} \]

At the time of exit the velocities of the charged particle in the \( x \) and \( z \) directions are

\[ u_x = 9.81 \times 638.55 \times 10^{-3} = 6.264 \text{ m/s} \]
\[ u_z = 1.0 \times 638.55 \times 10^{-3} = 0.639 \text{ m/s} \]

Thus, the exit velocity of the quartz particle is

\[ \vec{u} = 6.264\hat{u}_x - 0.639\hat{u}_z \text{ m/s} \]

**EXERCISES**

6.7 A phosphate granule with a mass of 1.2 grams acquires a charge of \(-100 \text{ nC}\) on the vibrating feeder. The particle falls freely at the middle of the top edge of the parallel plates, which are held at a potential of 5 kV. If the plates are 1.5 m in length, find the separation between the plates so that the phosphate granule barely touches the bottom of the positively charged plate. What is the velocity of the charged particle at the time of its exit?

6.8 The mass of a phosphate granule in Exercise 6.7 varies from 0.5 gram to 2.5 grams. The charge acquired by each granule also varies from \(-80 \text{ nC}\) to \(-120 \text{ nC}\). What must be the minimum separation between the plates if the particles fall freely at the middle of the top edge of the parallel plates?

6.6 Electrostatic Generator

An electrostatic generator conceived by Lord Kelvin was put in practice by Robert J. Van de Graaff and since then has been called the Van de Graaff generator. It consists of a hollow spherical conductor (dome) supported on an insulated hollow column, as shown in Figure 6.7a. A belt passes over the pulleys. The lower pulley is driven by a motor, and the upper is an idler. A number of sharp points projecting from a rod are maintained at a very high positive potential, and the air around the points becomes ionized. The positive ions are repelled from the sharp points and some of these ions attach themselves to the surface of the moving belt. A similar process takes place at the metal brush inside the dome. As the charge builds up, the potential of the dome rises. With the Van de Graaff generator a potential difference as high as several million volts can be realized. Its chief application is to accelerate the charged particles to acquire high kinetic energies, which are then used in atom-smashing experiments.

In order to understand this generator’s basic principle of operation, consider a hollow, uncharged, conducting sphere (dome) with a small opening, as shown in Figure 6.7b. Let us now introduce a positively charged small sphere with charge \( q \) through the opening into the cavity. As soon as the equilibrium state is reached, the inner surface of the dome acquires a net negative charge, while a positive charge \( q \) is induced on its outer surface. If the small sphere is now made to touch the inner surface of the dome, the positive charge