1) An analog source produces a baseband voltage signal $x(t)$ with bandwidth equal to 10 kHz. Assume that sample functions of $x(t)$ follow a probability density given by:

$$f_X(x) = \begin{cases} \frac{1}{2} e^{-x/2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The source output is quantized according to the rule:

$$y_i = \begin{cases} i - 0.5, & i - 1 \leq x < i, \text{ for } 1 \leq i \leq 5 \\ 6.0, & x \geq 5.0, i = 6 \end{cases}$$

Each quantized level $y_i$ is denoted as a discrete source symbol $g_i$ for $i = 1, 2, \ldots, 6$.

1.a. Determine the average quantization distortion.
1.b. What is the minimum required bit rate of a fixed-length encoder for the quantizer output?
1.c. Determine the discrete source average entropy.
1.d. Determine the minimum source encoder bit rate. Compare to part b.
1.e. Design a Huffman code for $\{g_i\}$ and calculate:
   i) the average codeword length
   ii) the code efficiency
   iii) the encoder minimum bit rate (Compare to parts b and d)

2) A message signal $x(t) = A_m \cos(2\pi f_m t)$ is sampled at a rate $f_s = 8 f_m$. A 2nd order (two-tap FIR) linear predictor based on the minimum mean squared error is to be employed to encode this signal using 16-level DPCM. Assume the autocorrelation function of $x(nT_s)$ to be denoted for short by $x(n)$ - can be estimated using the relation:

$$\hat{R}_{xx}(m) \approx \frac{1}{N} \sum_{i=0}^{N-m} x(i)x(i+m)$$

2.a. What is the DPCM encoder bit rate?
2.b. Derive conditions on the optimum predictor taps.
2.c. Assuming $N = 4$, determine the optimum coefficients of the predictor.

3) A discrete memoryless information source has an alphabet consisting of three symbols $a_1, a_2, a_3$ with respective probabilities 0.3, 0.6, 0.1. Symbols are emitted at a rate of 2000 symbols per second. A binary Huffman code is designed to represent the source output (call this Code A). Another binary Huffman code is designed to represent the 2nd extension of the source output (call this Code B).
3.a. Determine the codewords of Code A.
3.b. Determine the code efficiency of Code A.
3.c. Determine the bit rate of encoder A.
3.d. Determine the codewords of Code B.
3.e. Determine the code efficiency of Code B.
3.f. Determine the bit rate of encoder B.

4) A 4-symbol baseband digital communication system uses the waveforms below to represent its symbols, each of duration $T$. Assume that transmission is corrupted by AWGN with zero mean and $N_0/2$ power spectral density.

$$
\begin{align*}
    s_1(t) &= \begin{cases} 
        1, & 0 \leq t \leq T/2 \\
        0, & \text{elsewhere}
    \end{cases} \\
    s_2(t) &= \begin{cases} 
        1, & T/2 \leq t \leq T \\
        0, & \text{elsewhere}
    \end{cases} \\
    s_3(t) &= \begin{cases} 
        1, & 0 \leq t \leq T/2 \\
        -1, & T/2 \leq t \leq T
    \end{cases} \\
    s_4(t) &= \begin{cases} 
        1, & T/2 \leq t \leq T
    \end{cases}
\end{align*}
$$

4.a. Use Gram-Schmidt orthogonalization to determine two basis functions $\phi_1(t)$ and $\phi_2(t)$ to represent the signals above.
4.b. Sketch the signal space representation of the four signals.
4.c. Determine and sketch the noise-free outputs of two filters matched to $\phi_1(t)$ and $\phi_2(t)$ when $s_3(t)$ is transmitted.

5) An $M$-ary amplitude shift keying digital communication system transmits every $T$ seconds a signal of the form:

$$
s_i(t) = A_i \cos(2\pi f_i t), \quad A_i = 2i - M + 1, \quad i = 0, 1, \ldots, M - 1
$$

5.a. Determine and sketch the power spectral density of the transmitted ASK signal.
5.b. How does the PSD depend on $M$?

6) An 8-symbol digital communication system transmits every $T$ seconds a signal of the form:

$$
s_i(t) = A_i \cos(2\pi f_i t) + B_i \cos(2\pi f_2 t)
$$

where $f_2 = f_1 + 1/2T$. Assume $A_i$ takes values from the set $\{\pm d, \pm 3d\}$, while $B_i = \pm d$, where $d$ is a positive constant. The transmitted signal is corrupted by an zero-mean AWGN process with an autocorrelation function equal to $R_n(\tau) = \frac{N_0}{2} \delta(\tau)$. Assume equally-likely symbols.
6.a. Completely specify the optimum demodulator.
6.b. Determine the minimum average symbol probability of error.