**Problem 1.** A small ferry boat is 4.00 m wide and 6.00 m long. When a loaded truck pulls onto it, the boat sinks an additional 4.00 cm into the river. What is the weight of the truck?

**Solution:** The boat sinks until the weight of the additional water displaced equals the weight of the truck. Thus,

\[
W_{\text{truck}} = \rho_{\text{water}} (\Delta V) g = \left(10^3 \text{ kg/m}^3\right) \left[(4.00 \text{ m})(6.00 \text{ m})(4.00 \times 10^{-2} \text{ m})\right] \left(9.80 \text{ m/s}^2\right),
\]

or \[W_{\text{truck}} = 9.41 \times 10^3 \text{ N} = 9.41 \text{ kN}\]

**Problem 2.** The density of ice is 920 kg/m³, and that of sea water is 1030 kg/m³. What fraction of the total volume of an iceberg is exposed?

**Solution:** When the iceberg floats, the weight of the displaced water must equal the weight of the ice. Thus,

\[
\rho_{\text{water}} V_{\text{water}} g = \rho_{\text{ice}} V_{\text{ice}} g,
\]

or

\[
\frac{V_{\text{water}}}{V_{\text{ice}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}}.\]

The volume of the displaced water equals the volume of ice submerged, so the volume of ice exposed is \[V_{\text{exposed}} = V_{\text{ice}} - V_{\text{submerged}} = V_{\text{ice}} - V_{\text{water}}\] and the fraction of the ice exposed is

\[
\frac{V_{\text{exposed}}}{V_{\text{ice}}} = 1 - \frac{V_{\text{water}}}{V_{\text{ice}}} = 1 - \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = 1 - \frac{920 \text{ kg/m}^3}{1030 \text{ kg/m}^3} = 0.107, \text{ or } 10.7\%\]

**Problem 3.** A 10.0-kg block of metal is suspended from a scale and immersed in water as in the figure below. The dimensions of the block are 12.0 cm x 10.0 cm x 10.0 cm. The 12.0-cm dimension is vertical, and the top of the block is 5.00 cm below the surface of the water. (a) What are the forces exerted by the water on the top and bottom of the block? (Take \( P_0 = 1.013 \times 10^5 \text{ N/m}^2 \).) (b) What is the reading of the spring scale? (c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.
Solution:

(a) The absolute pressure at the level of the top of the block is

\[
P_{\text{top}} = P_0 + \rho_{\text{water}} g h_{\text{top}}
\]

\[
= 1.0130 \times 10^5 \text{ Pa} + \left(10^3 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) \left(5.00 \times 10^{-2} \text{ m}\right)
\]

\[
= 1.0179 \times 10^5 \text{ Pa}
\]

and that at the level of the bottom of the block is

\[
P_{\text{bottom}} = P_0 + \rho_{\text{water}} g h_{\text{bottom}}
\]

\[
= 1.0130 \times 10^5 \text{ Pa} + \left(10^3 \text{ kg/m}^3\right) \left(9.80 \text{ m/s}^2\right) \left(17.0 \times 10^{-2} \text{ m}\right)
\]

\[
= 1.0297 \times 10^5 \text{ Pa}
\]

Thus, the downward force exerted on the top by the water is

\[
F_{\text{top}} = P_{\text{top}} A = \left(1.0179 \times 10^5 \text{ Pa}\right) \left(0.100 \text{ m}\right)^2 = 1017.9 \text{ N}
\]

and the upward force the water exerts on the bottom of the block is

\[
F_{\text{bot}} = P_{\text{bot}} A = \left(1.0297 \times 10^5 \text{ Pa}\right) \left(0.100 \text{ m}\right)^2 = 1029.7 \text{ N}
\]

(b) The scale reading equals the tension, \(T\), in the cord supporting the block. Since the block is in equilibrium, \(\Sigma F_y = T + F_{\text{bot}} - F_{\text{top}} - mg = 0\), or

\[
T = (10.0 \text{ kg}) \left(9.80 \text{ m/s}^2\right) - (1029.7 - 1017.9) \text{ N} = 86.2 \text{ N}
\]

(c) From Archimedes's principle, the buoyant force on the block equals the weight of the displaced water. Thus,

\[
B = \left(\rho_{\text{water}} V_{\text{block}}\right) g
\]

\[
= \left(10^3 \text{ kg/m}^3\right) \left(0.100 \text{ m}\right)^2 \left(0.120 \text{ m}\right) \left(9.80 \text{ m/s}^2\right) = 11.8 \text{ N}
\]

From part (a), \(F_{\text{bot}} - F_{\text{top}} = (1029.7 - 1017.9) \text{ N} = 11.8 \text{ N}\), which is the same as the buoyant force found above.
Problem 4. A thin spherical shell of mass 0.400 kg and diameter 0.200 m is filled with alcohol ($\rho = 806 \text{ kg/m}^3$). It is then released from rest at the bottom of a pool of water. Find the acceleration of the alcohol-filled shell as it starts to rise toward the surface of the water.

Solution: The volume of the shell is $V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \left( \frac{0.200 \text{ m}}{2} \right)^3 = 4.19 \times 10^{-3} \text{ m}^3$, so the mass of alcohol filling the shell is

$$m_a = \rho_a V = (806 \text{ kg/m}^3) \left( 4.19 \times 10^{-3} \text{ m}^3 \right) = 3.38 \text{ kg},$$

and the mass of the filled shell is

$$m = m_{\text{shell}} + m_a = (0.400 + 3.38) \text{ kg} = 3.78 \text{ kg}.$$

The buoyant force exerted on the shell by the water is

$$B = (\rho_{\text{water}} V)g = \left( 10^3 \text{ kg/m}^3 \right) \left( 4.19 \times 10^{-3} \text{ m}^3 \right) \left( 9.80 \text{ m/s}^2 \right) = 41.1 \text{ N},$$

and the upward acceleration is

$$a_y = \frac{\Sigma F_y}{m} = \frac{B - mg}{m} = \frac{B}{m} - g = \frac{41.1 \text{ N}}{3.78 \text{ kg}} - 9.80 \text{ m/s}^2 = 1.07 \text{ m/s}^2.$$

Problem 5. A rectangular air mattress is 2.0 m long, 0.50 m wide, and 0.08 m thick. If it has a mass of 2.0 kg, what additional mass can it support in water?

Solution: When the mattress is fully submerged, the buoyant force exerted by the water (and hence the total weight that can be supported) is

$$B = (\rho_{\text{water}} V)g$$

$$= \left( 10^3 \text{ kg/m}^3 \right) \left[ (2.0 \text{ m})(0.50 \text{ m})(0.080 \text{ m}) \right] (9.80 \text{ m/s}^2) = 7.8 \times 10^2 \text{ N}$$

Thus, the total mass that can be supported is

$$m_{\text{total}} = \frac{B}{g} = \frac{7.8 \times 10^2 \text{ N}}{9.80 \text{ m/s}^2} = 80 \text{ kg}.$$

The additional mass that can be placed on the mattress is then

$$m_{\text{additional}} = m_{\text{total}} - m_{\text{mattress}} = 80 \text{ kg} - 2.0 \text{ kg} = 78 \text{ kg}.$$
Problem 6. An empty rubber balloon has a mass of 0.0120 kg. The balloon is filled with helium at a density of 0.181 kg/m$^3$. At this density the balloon has a radius of 0.500 m. If the filled balloon is fastened to a vertical line, what is the tension in the line?

Solution: The balloon is in equilibrium under the action of three forces. These are the buoyant force, $B$, the total weight, $W$, of the balloon and the helium, and the tension $T$ in the string. Hence,

$$\Sigma F_y = B - \left( m_{\text{balloon}} + m_{\text{helium}} \right) g - T = 0,$$

or

$$T = B - \left( m_{\text{balloon}} + m_{\text{helium}} \right) g$$

The buoyant force is

$$B = \left( \rho_{\text{air}} V_{\text{balloon}} \right) g$$

and $m_{\text{helium}} = \rho_{\text{helium}} V_{\text{balloon}}$, where $V_{\text{balloon}} = \frac{4\pi r^3}{3}$

Thus,

$$T = \left( \rho_{\text{air}} - \rho_{\text{helium}} \right) g \left( \frac{r^3}{3} \right) - m_{\text{balloon}} g$$

$$= \left[ (1.29 - 0.181) \text{ kg/m}^3 \right] \left( 9.80 \text{ m/s}^2 \right) \left( \frac{4\pi}{3} \right) (0.500 \text{ m})^3$$

$$- (0.0120 \text{ kg})(9.80 \text{ m/s}^2)$$

or

$$T = 5.57 \text{ N}$$

Problem 7. (a) Calculate the mass flow rate (in grams per second) of blood ($\rho = 1.0$ g/cm$^3$) in an aorta with a cross-sectional area of 2.0 cm$^2$ if the flow speed is 40 cm/s. (b) Assume that the aorta branches to form a large number of capillaries with a combined cross-sectional area of $3.0 \times 10^3$ cm$^2$. What is the flow speed in the capillaries?

Solution: (a) The volume flow rate is $A v$, and the mass flow rate is

$$\rho A v = \left( 1.0 \text{ g/cm}^3 \right) (2.0 \text{ cm}^2)(40 \text{ cm/s}) = 80 \text{ g/s}$$

(b) From the equation of continuity, the speed in the capillaries is

$$v_{\text{capillaries}} = \left( \frac{A_{\text{aorta}}}{A_{\text{capillaries}}} \right) v_{\text{aorta}} = \left( \frac{2.0 \text{ cm}^2}{3.0 \times 10^3 \text{ cm}^2} \right)(40 \text{ cm/s}),$$

or

$$v_{\text{capillaries}} = 2.7 \times 10^{-2} \text{ cm/s} = 0.27 \text{ mm/s}$$
Problem 8. A large storage tank, open to the atmosphere at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is $2.50 \times 10^{-3}$ $\text{m}^3/\text{min}$, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

Solution: (a) Apply Bernoulli's equation with point 1 at the open top of the tank and point 2 at the opening of the hole. Then, $P_1 = P_2 = P_{\text{atm}}$, and we assume $v_1 \approx 0$. This gives $\frac{1}{2} \rho v_2^2 + \rho g y_2 = \rho g y_1$, or

$$v_2 = \sqrt{2 g (y_1 - y_2)} = \sqrt{2 (9.80 \text{ m/s}^2) (16.0 \text{ m})} = 17.7 \text{ m/s}$$

(b) The area of the hole is found from

$$A_2 = \frac{\text{flow rate}}{v_2} = \frac{2.50 \times 10^{-3} \text{ m}^3/\text{min}}{17.7 \text{ m/s}} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 2.35 \times 10^{-6} \text{ m}^2$$

The diameter is then

$$d_2 = \frac{4 A_2}{\pi} = \frac{4 \left(2.35 \times 10^{-6} \text{ m}^2\right)}{\pi} = 1.73 \times 10^{-3} \text{ m} = 1.73 \text{ mm}$$

Problem 9. A sample of an unknown material appears to weigh 300 N in air and 200 N when immersed in alcohol of specific gravity 0.700. What are (a) the volume and (b) the density of the material?

Solution: (a) The buoyant force is the difference between the weight in air and the apparent weight when immersed in the alcohol, or $B = 300 \text{ N} - 200 \text{ N} = 100 \text{ N}$. But, from Archimedes's principle, this is also the weight of the displaced alcohol, so $B = (\rho_{\text{alcohol}} V) g$. Since the sample is fully submerged, the volume of the displaced alcohol is the same as the volume of the sample. This volume is

$$V = \frac{B}{\rho_{\text{alcohol}} g} = \frac{100 \text{ N}}{(700 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 1.46 \times 10^{-2} \text{ m}^3$$

(b) The mass of the sample is $m = \frac{\text{weight in air}}{g} = \frac{300 \text{ N}}{9.80 \text{ m/s}^2} = 30.6 \text{ kg}$, and its density is $\rho = \frac{m}{V} = \frac{30.6 \text{ kg}}{1.46 \times 10^{-2} \text{ m}^3} = 2.10 \times 10^3 \text{ kg/m}^3$. 
Problem 10. An object weighing 300 N in air is immersed in water after being tied to a string connected to a balance. The scale now reads 265 N. Immersed in oil, the object appears to weigh 275 N. Find (a) the density of the object and (b) the density of the oil

Solution: The difference between the weight in air and the apparent weight when immersed is the buoyant force exerted on the object by the fluid.

(a) The mass of the object is \( m = \frac{\text{weight in air}}{g} = \frac{300 \text{ N}}{9.80 \text{ m/s}^2} = 30.6 \text{ kg} \). The buoyant force when immersed in water is the weight of a volume of water equal to the volume of the object, or \( B_w = (\rho_w V)g \). Thus, the volume of the object is

\[
V = \frac{B_w}{\rho_w g} = \frac{300 \text{ N} - 265 \text{ N}}{(10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.57 \times 10^{-3} \text{ m}^3,
\]

and its density is \( \rho_{\text{object}} = \frac{m}{V} = \frac{30.6 \text{ kg}}{3.57 \times 10^{-3} \text{ m}^3} = 8.57 \times 10^3 \text{ kg/m}^3 \)

(b) The buoyant force when immersed in oil is equal to the weight of a volume \( V = 3.57 \times 10^{-3} \text{ m}^3 \) of oil. Hence, \( B_{\text{oil}} = (\rho_{\text{oil}} V)g \), or the density of the oil is

\[
\rho_{\text{oil}} = \frac{B_{\text{oil}}}{Vg} = \frac{300 \text{ N} - 275 \text{ N}}{(3.57 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2)} = 714 \text{ kg/m}^3
\]