Problem 1. If a current of 80.0 mA exists in a metal wire, how many electrons flow past a given cross section of the wire in 10.0 min? Sketch the directions of the current and the electrons’ motion.

Solution: The charge that moves past the cross section is \( \Delta Q = \mathcal{A} \Delta t \), and the number of electrons is

\[
n = \frac{\Delta Q}{|q|} = \frac{\mathcal{A} \Delta t}{|q|} = \frac{(80 \, \text{mA} \times 10^{-3} \, \text{C} / \text{s}) \left[ (10 \, \text{m in}) \left( 60 \, \text{m in} / \text{s} \right) \right]}{1.60 \times 10^{-19} \, \text{C}} = 3.00 \times 10^{20} \text{ electrons}
\]

The negatively charged electrons move in the direction opposite to the conventional current flow.

Problem 2. A small sphere that carries a charge \( q \) is whirled in a circle at the end of an insulating string. The angular frequency of rotation is \( \omega \). What average current does this rotating charge represent?

Solution: The period of revolution for the sphere is \( T = \frac{2\pi}{\omega} \), and the average current represented by this revolving charge is

\[
I = \frac{q}{T} = \frac{q \omega}{2\pi}
\]

Problem 3. In a particular television picture tube, the measured beam current is 60.0 \( \mu \text{A} \). How many electrons strike the screen every second?

Solution: \( \Delta Q = \mathcal{A} \Delta t \) and the number of electrons is

\[
n = \frac{\Delta Q}{|q|} = \frac{\mathcal{A} \Delta t}{|q|} = \frac{(60 \, \text{mA} \times 10^{-6} \, \text{C} / \text{s}) (1.00 \, \text{s})}{1.60 \times 10^{-19} \, \text{C}} = 3.75 \times 10^{14} \text{ electrons}
\]

Problem 4. A total charge of 6.0 mC passes through a cross-sectional area of a wire in 2.0 s. What is the current in the wire?

Solution: The current is

\[
I = \frac{\Delta Q}{\Delta t} = \frac{6.0 \times 10^{-3} \, \text{C}}{2 \, \text{s}} = 3.0 \times 10^{-3} \, \text{A} = 3 \, \text{mA}
\]
**Problem 5.** In the Bohr model of the hydrogen atom, an electron in the lowest energy state moves at a speed of $2.19 \times 10^6$ m/s in a circular path having a radius of $5.29 \times 10^{-11}$ m. What is the effective current associated with this orbiting electron?

**Solution:** The period of the electron in its orbit is $T = \frac{2\pi r}{v}$, and the current represented by the orbiting electron is

\[
I = \frac{\Delta Q}{\Delta t} = \frac{\frac{qd}{T}}{2\pi r} = \frac{\left(2.19 \times 10^6 \text{ m/s}\right) \left(1.60 \times 10^{-19} \text{ C}\right)}{2\pi \left(5.29 \times 10^{-11} \text{ m}\right)} = 1.05 \times 10^{-3} \text{ C/s} = 1.05 \text{ mA}
\]

**Problem 6.** If $3.25 \times 10^{-3}$ kg of gold is deposited on the negative electrode of an electrolytic cell in a period of 2.78 h, what is the current through the cell in this period? Assume that the gold ions carry one elementary unit of positive charge.

**Solution:** The mass of a single gold atom is

\[
m_{\text{atom}} = \frac{M}{N_A} = \frac{197 \text{ g/mol}}{6.02 \times 10^{23} \text{ atom/mol}} = 3.27 \times 10^{-22} \text{ g} = 3.27 \times 10^{-25} \text{ kg}
\]

The number of atoms deposited, and hence the number of ions moving to the negative electrode, is

\[
n = \frac{m}{m_{\text{atom}}} = \frac{3.25 \times 10^{-3} \text{ kg}}{3.27 \times 10^{-25} \text{ kg}} = 9.93 \times 10^{21}
\]

Thus, the current in the cell is

\[
I = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t} = \frac{\left(9.93 \times 10^{21}\right) \left(1.60 \times 10^{-19} \text{ C}\right)}{\left(2.78 \text{ h}\right) \left(3600 \text{ s/h}\right)} = 0.159 \text{ A} = 159 \text{ mA}
\]
Problem 7. A 200-km-long high-voltage transmission line 2.0 cm in diameter carries a steady current of 1 000 A. If the conductor is copper with a free charge density of \(8.5 \times 10^{28}\) electrons per cubic meter, how long (in years) does it take one electron to travel the full length of the cable?

Solution: The drift speed of electrons in the line is \(v_d = \frac{I}{n q A} = \frac{I}{n q A \pi d / 4}\), or

\[
v_d = \frac{4(1000 \text{ A})}{(8.5 \times 10^{28} / \text{m}^3)(1.60 \times 10^{-19} \text{ C} \pi (0.020 \text{ m})^2)} = 2.3 \times 10^{-4} \text{ m/s}
\]

The time to travel the length of the 200-km line is then

\[
\Delta t = \frac{L}{v_d} = \frac{200 \times 10^3 \text{ m}}{2.34 \times 10^{-4} \text{ m/s} \times 3.156 \times 10^7 \text{ s}} = 27 \text{ yr}
\]

Problem 8. An aluminum wire with a cross-sectional area of \(4.0 \times 10^{-6} \text{ m}^2\) carries a current of 5.0 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.7 g/cm³. (Assume that one electron is supplied by each atom.)

Solution: Assuming that, on average, each aluminum atom contributes one electron, the density of charge carriers is the same as the number of atoms per cubic meter. This is

\[
n = \frac{\text{density}}{\text{mass per atom}} = \frac{\rho}{M / N_A} = \frac{N_A \rho}{M},
\]

or

\[
n = \frac{6.02 \times 10^{23} / \text{mol}}{(2.7 \text{ g/cm}^3)(10^6 \text{ cm}^3 / 1 \text{ m}^3)} = 6.0 \times 10^{28} / \text{m}^3
\]

The drift speed of the electrons in the wire is then

\[
v_d = \frac{I}{n q A} = \frac{5 \text{ C/s}}{6.0 \times 10^{28} / \text{m}^3(1.60 \times 10^{-19} \text{ C})(4 \times 10^{-6} \text{ m}^2)} = 1.3 \times 10^{-4} \text{ m/s}
\]

Problem 9. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density and (b) electron drift velocity?

Solution: (a) The carrier density is determined by the physical characteristics of the wire, not the current in the wire. Hence, \(n\) is \underline{unaffected}.

(b) The drift velocity of the electrons is \(v_d = I / n q A\). Thus, the drift velocity is \underline{doubled} when the current is doubled.
Problem 10. A lightbulb has a resistance of 240 \( \Omega \) when operating at a voltage of 120 V. What is the current through the lightbulb?

Solution: 
\[
I = \frac{\Delta V}{R} = \frac{120 \text{ V}}{240 \Omega} = 0.500 \text{ A} = 500 \text{ mA}
\]

Problem 11. A person notices a mild shock if the current along a path through the thumb and index finger exceeds 80 \( \mu \text{A} \). Compare the maximum allowable voltage without shock across the thumb and index finger with a dry-skin resistance of \( 4.0 \times 10^5 \Omega \) and a wet-skin resistance of 2000 \( \Omega \).

Solution: 
\[
(I_{\text{max}})_{\text{dry}} = I_{\text{max}} R = \left(80 \times 10^{-6} \text{ A}\right) R
\]

Thus, if \( R = 4.0 \times 10^5 \Omega \), \( (\Delta V)_{\text{max}} = 32 \text{ V} \).

and if \( R = 2000 \Omega \), \( (\Delta V)_{\text{max}} = 0.16 \text{ V} \).

Problem 12. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of \( R = 0.500 \Omega \), and if all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?

Solution: The volume of the copper is
\[
V = \frac{m}{\text{density}} = \frac{1.00 \times 10^{-3} \text{ kg}}{8.92 \times 10^3 \text{ kg/m}^3} = 1.12 \times 10^{-7} \text{ m}^3
\]

Since, \( V = A \cdot L \), this gives \( A \cdot L = 1.12 \times 10^{-7} \text{ m}^3 \). (1)

(a) From \( R = \frac{\rho L}{A} \), we find that
\[
A = \left(\frac{\rho}{R}\right) L = \left(\frac{1.70 \times 10^{-8} \text{ \Omega m}}{0.500 \Omega}\right)L = \left(3.40 \times 10^{-8} \text{ m}\right)L.
\]

Inserting this expression for \( A \) into Equation 1 gives
\[
\left(3.40 \times 10^{-8} \text{ m}\right)L = 1.12 \times 10^{-7} \text{ m}^3, \text{ which yields } L = 1.82 \text{ m}
\]

(b) From equation (1), \( A = \frac{\pi d^2}{4} = \frac{1.12 \times 10^{-7} \text{ m}^3}{L} \), or
\[
d = \sqrt{\frac{4 \left(1.12 \times 10^{-7} \text{ m}^3\right)}{\pi L}} = \sqrt{\frac{4 \left(1.12 \times 10^{-7} \text{ m}^3\right)}{\pi (1.82 \text{ m})}}
\]

\[
d = 2.80 \times 10^{-4} \text{ m} = 0.280 \text{ mm}
\]
Problem 13. Calculate the diameter of a 2.0-cm length of tungsten filament in a small lightbulb if its resistance is 0.050 Ω.

Solution: From \( R = \frac{\rho L}{A} \), we obtain \( A = \frac{\pi d^2}{4} = \frac{\rho L}{R} \), or

\[
\frac{4 \rho L}{\pi R} = \left( \frac{5.6 \times 10^{-8} \Omega \cdot m}{2.0 \times 10^{-2} \text{ m}} \right) \pi (0.050 \Omega) = 1.7 \times 10^{-4} \text{ m} = 0.17 \text{ mm}
\]

Problem 14. Eighteen-gauge wire has a diameter of 1.024 mm. Calculate the resistance of 15 m of 18-gauge copper wire at 20°C.

Solution: \( R = \frac{\rho L}{A} = \frac{\rho L}{\pi d^2 / 4} = \frac{4 \left( 1.7 \times 10^{-8} \Omega \cdot m \right) (15 \text{ m})}{\pi (1.024 \times 10^{-3} \text{ m})^2} = 0.31 \Omega \)

Problem 15. A potential difference of 12 V is found to produce a current of 0.40 A in a 3.2-m length of wire with a uniform radius of 0.40 cm. What is (a) the resistance of the wire and (b) the resistivity of the wire?

Solution: (a) \( R = \frac{\Delta V}{I} = \frac{12 \text{ V}}{0.40 \text{ A}} = 30 \Omega \)

(b) From, \( R = \frac{\rho L}{A} \),

\[
\rho = \frac{R \cdot A}{L} = \left( 30 \Omega \right) \left[ \frac{\pi \left( 0.40 \times 10^{-2} \text{ m} \right)^2}{32 \text{ m}} \right] = 4.7 \times 10^{-4} \Omega \cdot \text{m}
\]

Problem 16. A length \( L_0 \) of copper wire has a resistance \( R_0 \). The wire is cut into three pieces of equal length. The pieces are then connected as parallel lengths between points \( A \) and \( B \). What resistance will this new “wire” of length \( L_0/3 \) have between points \( A \) and \( B' \)?

Solution: The new “wire” has length \( L = L_0 / 3 \) and cross-section \( A = 3 A_0 \). Thus, its resistance is

\[
R = \frac{\rho L}{A} = \frac{\rho \left( L_0 / 3 \right)}{3 A_0} = \frac{1}{9} \left( \frac{\rho L_0}{A_0} \right) = \frac{R_0}{9}
\]

Problem 17. A wire 50.0 m long and 2.00 mm in diameter is connected to a source with a potential difference of 9.11 V, and the current is found to be 36.0 A. Identify the metal of the wire.

Solution: The resistance is \( R = \frac{\Delta V}{I} = \frac{9.11 \text{ V}}{36.0 \text{ A}} = 0.253 \Omega \), so the resistivity of the metal is

\[
\rho = \frac{R \cdot A}{L} = \frac{R \left( \pi d^2 / 4 \right)}{L} = \frac{0.253 \Omega \pi \left( 2.00 \times 10^{-2} \text{ m} \right)^2}{4(50.0 \text{ m})} = 1.59 \times 10^{-8} \Omega \cdot \text{m}
\]

Thus, the metal is seen to be silver.
Problem 18. A 4.0-Ω resistor, an 8.0-Ω resistor, and a 12-Ω resistor are connected in series with a 24-V battery. What are (a) the equivalent resistance and (b) the current in each resistor? (c) Repeat for the case in which all three resistors are connected in parallel across the battery.

Solution: (a) \[ R_{eq} = R_1 + R_2 + R_3 = 4 \Omega + 8 \Omega + 12 \Omega = 24 \Omega \]

(b) The same current exists in all resistors in a series combination.
\[ I = \frac{\Delta V}{R_{eq}} = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A} \]

(c) If the three resistors were connected in parallel,
\[ R_{eq} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} = \left( \frac{1}{4 \Omega} + \frac{1}{8 \Omega} + \frac{1}{12 \Omega} \right)^{-1} = 2.18 \Omega \]

Resistors in parallel have the same potential difference across them, so
\[ I_1 = \frac{\Delta V}{R_1} = \frac{24 \text{ V}}{4 \Omega} = 6 \text{ A} , \quad I_2 = \frac{24 \text{ V}}{8 \Omega} = 3 \text{ A} , \quad \text{and} \quad I_3 = \frac{24 \text{ V}}{12 \Omega} = 2 \text{ A} \]

Problem 19. A 9.0-Ω resistor and a 6.0-Ω resistor are connected in series with a power supply. (a) The voltage drop across the 6.0-Ω resistor is measured to be 12 V. Find the voltage output of the power supply. (b) The two resistors are connected in parallel across a power supply, and the current through the 9.0-Ω resistor is found to be 0.25 A. Find the voltage setting of the power supply.

Solution: (a) The current through this series combination is
\[ I = \frac{(\Delta V)_{bc}}{R_{bc}} = \frac{12 \text{ V}}{6 \Omega} = 2 \text{ A} \]

Therefore, the terminal potential difference of the power supply is
\[ \Delta V = IR_{eq} = (2 \text{ A})(9 \Omega + 6 \Omega) = 30 \text{ V} \]

(b) When connected in parallel, the potential difference across either resistor is the voltage setting of the power supply. Thus,
\[ \Delta V = I_3 R_3 = (0.25 \text{ A})(9 \Omega) = 2.3 \text{ V} \]
Problem 20.  (a) Find the equivalent resistance between points \(a\) and \(b\) in the following figure. (b) Calculate the current in each resistor if a potential difference of 34.0 V is applied between points \(a\) and \(b\).

Solution:  
(a) The equivalent resistance of the two parallel resistors is

\[
R_p = \left( \frac{1}{7.00 \, \Omega} + \frac{1}{10.0 \, \Omega} \right)^{-1} = 4.12 \, \Omega
\]

Thus,

\[
R_{ab} = R_4 + R_p + R_5 = (4.00 + 4.12 + 9.00) \, \Omega = 17.1 \, \Omega
\]

(b) \[
I_{ab} = \frac{(\Delta V)_{ab}}{R_{ab}} = \frac{34.0 \, V}{17.1 \, \Omega} = 1.99 \, A , \text{ so } I_4 = I_5 = 1.99 \, A
\]

Also,

\[
(\Delta V)_p = I_p R_p = (1.99 \, A)(4.12 \, \Omega) = 8.18 \, V
\]

Then,

\[
I_7 = \frac{(\Delta V)_p}{R_7} = \frac{8.18 \, V}{7.00 \, \Omega} = 1.17 \, A
\]

and \[
I_0 = \frac{(\Delta V)_p}{R_{10}} = \frac{8.18 \, V}{10.0 \, \Omega} = 0.818 \, A
\]
Problem 21. Find the equivalent resistance of the circuit in figure shown.

Solution: The equivalent resistance of the parallel combination of three resistors is

\[
R_p = \left( \frac{1}{18 \, \Omega} + \frac{1}{9 \, \Omega} + \frac{1}{6 \, \Omega} \right)^{-1} = 3 \, \Omega
\]

Hence, the equivalent resistance of the circuit connected to the 30 V source is

\[
R_{eq} = R_{12} + R_p = 12 \, \Omega + 3 \, \Omega = 15 \, \Omega
\]

Problem 22. Find the equivalent resistance of the circuit in the figure shown

Solution: The rules for combining resistors in series and parallel are used to reduce the circuit to an equivalent resistance in the stages shown below. The result is \( R_{eq} = 9.8 \, \Omega \).
Problem 23. Consider the circuit shown in the following figure. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points $a$ and $b$.

Solution: Turn the circuit given in Figure 90° counterclockwise to observe that it is equivalent to that shown in Figure 1 below. This reduces, in stages, as shown in the following figures.

![Figure 1](image1.png)

![Figure 2](image2.png)

![Figure 3](image3.png)

![Figure 4](image4.png)

From Figure 4,

$$I = \frac{\Delta V}{R} = \frac{25.0 \text{ V}}{12.9 \Omega} = 1.93 \text{ A}$$

(b) From Figure 3,

$$\Delta V_{ba} = IR_{20}$$

$$= (1.93 \text{ A})(2.94 \Omega) = 5.68 \text{ V}$$

(a) From Figures 1 and 2, the current through the 20 Ω resistor is

$$I_{20} = \frac{\Delta V_{ba}}{R_{20}} = \frac{5.68 \text{ V}}{25 \Omega} = 0.227 \text{ A}$$
Problem 24. Two resistors, $A$ and $B$, are connected in parallel across a 6.0-V battery. The current through $B$ is found to be 2.0 A. When the two resistors are connected in series to the 6.0-V battery, a voltmeter connected across resistor $A$ measures a voltage of 4.0 V. Find the resistances of $A$ and $B$.

Solution: First, consider the parallel case. The resistance of resistor $B$ is

$$R_B = \frac{(\Delta V)_B}{I_B} = \frac{6 \Omega \cdot 6 \Omega}{2 \ \text{A}} = 3 \ \Omega$$

In the series combination, the potential difference across $B$ is given by

$$\Delta V_B = \Delta V_{\text{battery}} - (\Delta V)_A = 6 \Omega - 4 \Omega = 2 \Omega$$

The current through the series combination is then

$$I_S = \frac{(\Delta V)_B}{R_B} = \frac{2 \Omega}{3 \Omega} = \frac{2}{3} \ \text{A},$$

and the resistance of resistor $A$ is $R_A = \frac{(\Delta V)_A}{I_A} = \frac{4 \Omega \cdot 2}{3 \text{A}} = 6 \ \Omega$.

Problem 25. The resistance between terminals $a$ and $b$ in the following figure is 75 $\Omega$. If the resistors labeled $R$ have the same value, determine $R$.

![Diagram of circuit](image)

Solution: The equivalent resistance is $R_{eq} = R + R_P$, where $R_P$ is the total resistance of the three parallel branches;

$$R_P = \left( \frac{1}{120 \ \Omega} + \frac{1}{40 \ \Omega} + \frac{1}{R + 5 \ \Omega} \right)^{-1} = \left( \frac{1}{30 \ \Omega} + \frac{1}{R + 5 \ \Omega} \right)^{-1} = \frac{30 \ \Omega \cdot (R + 5 \ \Omega)}{R + 35 \ \Omega}$$

Thus, $75 \ \Omega = R + \frac{(30 \ \Omega)(R + 5 \ \Omega)}{R + 35 \ \Omega} = \frac{R^2 + (65 \ \Omega)R + 150 \ \Omega^2}{R + 35 \ \Omega}$,

which reduces to $R^2 - (10 \ \Omega)R + 2475 \ \Omega^2 = 0$ or $(R - 55 \ \Omega)(R + 45 \ \Omega) = 0$.

Only the positive solution is physically acceptable, so $R = 55 \ \Omega$.
Problem 26. Find the current in the 12-Ω resistor in the following figure

**Solution:** The resistors in the circuit can be combined in the stages shown below to yield an equivalent resistance of \( R_{eq} = \left( \frac{63}{11} \right) \Omega \).

From Figure 5, \( I = \frac{(\Delta V)_{ad}}{R_{ad}} = \frac{18 \text{ V}}{\left( \frac{63}{11} \right) \Omega} = 3.14 \text{ A} \).

Then, from Figure 4, \( \Delta V_{ad} = IR_{ad} = (3.14 \text{ A}) \left( \frac{30}{11} \Omega \right) = 8.57 \text{ V} \).

Now, look at Figure 2 and observe that

\[ I_2 = \frac{(\Delta V)_{bd}}{3 \Omega + 2 \Omega + 5 \Omega} = \frac{8.57 \text{ V}}{10 \Omega} = 0.86 \text{ A}, \]

so \( \Delta V_{bd} = I_2 R_{bd} = (0.86 \text{ A})(3 \Omega) = 2.58 \text{ V} \).

Finally, from Figure 1, \( I_2 = \frac{(\Delta V)_{bd}}{R_{12}} = \frac{5.14 \text{ V}}{12 \Omega} = 0.43 \text{ A} \).