Design of Optimum Gain Pyramidal Horn with Improved Formulas Using Particle Swarm Optimization

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Received 5 June 2006; accepted 29 August 2006

ABSTRACT: In this article, a new method to design an optimum gain horn antenna is presented. This method is based on improving existing design equations along with the use of particle swarm optimization technique. The obtained results are more accurate than those available in the literature since no path length error approximation is involved. Improved horn dimensions, for different 15 designs, are obtained which give the desired gain almost exactly. © 2007 Wiley Periodicals, Inc. *Int J RF and Microwave CAE* 17: 505–511, 2007.

Keywords: horn antenna; particle swarm optimization

I. INTRODUCTION

Optimum gain pyramidal horns are widely used as standard gain antennas since their gains can be calculated to within 0.1 dB using their known dimensions [1, 2]. An optimum gain pyramidal horn antenna is one that produces maximum gain for given E - and H -plane slant lengths. Several methods [1–8] have been presented and used to design a pyramidal horn with optimum gain. Each method has its own accuracy, limitations, and computational complexity. One common feature in [1–8] is the use of the expressions that give optimum gain that are derived from an approximate formula (the first two terms of binomial series expansion) for the path length error that gives rise to aperture quadratic phase error [1]. The method described in [1] is only valid for long, high-gain horns, and uses trial and error (iterative technique). The method proposed in [2] avoids the application of iterative techniques, and is not restricted to high-gain

horn design. However, the actual gain results do not agree well with the desired gain results. Also, the horn problem was formulated in terms of a fourth degree polynomial that was analytically solved for the narrow aperture dimension. The methods described in [3–5] improved the design equations given in [2] using curve fitting. The method proposed in [6] gives more accurate results than the previous methods, but it needs a large number of inputs and outputs to develop and train a neural network. Recently, in [7, 8], the differential evolution technique and the particle swarm optimization method were used to derive new expressions for the narrow and wide aperture dimensions, respectively.

In this article, improved formulas for the relationship between the optimum gain horn aperture size and lengths are derived using the more accurate gain expression given in [9], which does not involve the approximate path length error. These formulas are then used along with the particle swarm optimization technique [10–13] to find directly the physical dimensions of the horn. Our results are compared to those obtained using other techniques [1–8] and shown to give the desired gain exactly.

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DOI 10.1002/mmce.20245
Published online 22 June 2007 in Wiley InterScience (www.interscience.wiley.com).

II. PYRAMIDAL HORN DESIGN

Consider the geometry of a pyramidal horn antenna with the E -plane (B, B_1, L_E, L_1, P_E) and H -plane (A, A_1, L_H, L_2, P_H) cross-sectional views as shown in Figure 1. To design a pyramidal horn, one usually knows the desired gain G , the wavelength λ , and the inner

Figure 1. (a) Pyramidal horn antenna (b) E -plane view (c) H -plane view.

dimensions A and B of the feeding rectangular waveguide. The other parameters ($B_1, L_E, L_1, P_E, A_1, L_H, L_2$, and P_H) are then determined using a specific design method.

A horn is said to be optimum when the aperture dimensions are adjusted to give maximum gain for given slant lengths in the E - and H -planes. According to [1], this occurs when the following relations are satisfied:

$$B_1 = \sqrt{2\lambda L_1} \quad (1)$$

$$A_1 = \sqrt{3\lambda L_2} \quad (2)$$

This means that optimum gain is obtained when the maximum aperture phase deviations in the E - and H -planes are 0.25λ and 0.375λ , respectively [1]. The two equations above were obtained by means of curve fitting for (L_1, B_1) and (L_2, A_1) that give the maximum gain. However, it should be emphasized that the expressions used to derive eqs. (1) and (2) were not exact since the path length error, which gives rise to the quadratic phase error, was approximated by the first two terms of a binomial series expansion of the exact expression [1]. Such an approximation was done to simplify the derivation, which led to closed form expressions for the electric and magnetic fields in terms of cosine and sine Fresnel integrals. In most cases, it yields to very good results for long high gain horns. The above approximate relations were used in all the design methods presented in [1–8]. Here, new improved formulas will be presented as explained below.

In [9], an expression for the gain of pyramidal horn antenna was derived without approximating the path length error. The expression is given by:

$$G = \frac{32A_1B_1}{\pi\lambda^2} (L_e(S))(L_h(T)) \quad (3)$$

$$L_e(S) = \left| \frac{2}{B_1} \int_0^{B_1/2} e^{-jKL_1 [\sqrt{1+(y/L_1)^2} - 1]} dy \right|^2 \quad (4)$$

$$L_h(T) = \left| \frac{\pi}{A_1} \int_0^{A_1/2} \cos\left(\frac{\pi x}{A_1}\right) e^{-jKL_2 [\sqrt{1+(y/L_2)^2} - 1]} dx \right|^2 \quad (5)$$

It is interesting to note that the above expression was used in [2–8] to find the gain of the designed

Figure 2. *E*-plane normalized gain of pyramidal horn as a function of B_1 for different lengths of L_1 . [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

pyramidal horns, although the design itself involved the use of the approximate eqs. (1) and (2).

Now, using eq. (3), the *E*-plane normalized gain G_n is plotted versus B_1 for different lengths of L_1 , as shown in Figure 2. The normalized *E*-plane gain G_n is defined as:

$$G_n = \frac{G\pi\lambda^2}{32A_1L_h(T)} \quad (6)$$

The integrals in eqs. (4) and (5) are evaluated numerically within an error of 10^{-9} using an adaptive Gauss/Lobatto quadrature rule. If the values of B_1 (in λ) which correspond to the maximum gains in Figure 2 are plotted versus their corresponding values of L_1 (in λ), the following relation can be obtained using curve fitting for 25 points,

$$B_1 = \sqrt{2.1060\lambda L_1} \quad (7)$$

This corresponds to a maximum phase deviation in the *E*-plane of 0.2633λ .

Figure 3 shows the differences between eqs. (1) and (7) as compared to those obtained directly from eq. (3) using the peaks in Figure 2. It can be seen that eq. (7) gives better results for B_1 than eq. (1).

Similarly, Figure 4 shows the *H*-plane normalized gain versus A_1 for different lengths of L_2 as obtained using eq. (3). Here, the *H*-plane normalized gain G_n is defined as:

$$G_n = \frac{G\pi\lambda^2}{32B_1L_c(S)} \quad (8)$$

If the values of A_1 (in λ) which correspond to the maximum gains in Figure 4 are plotted versus their

Figure 3. B_1 versus L_1 obtained using eqs. (1) and (7) compared to values obtained from (3). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

corresponding values of L_2 (in λ), the following equation can be obtained using curve fitting for 25 points:

$$A_1 = \sqrt{3.1831\lambda L_2} \quad (9)$$

This corresponds to maximum phase deviation in the *H*-plane of 0.39789λ . Figure 5 shows the differences between eqs. (2) and (9) as compared to those obtained directly from eq. (3) using the peaks in Figure 4. It can be seen that eq. (9) gives better results for B_1 than eq. (2).

Recently, in [14], the phases that correspond to the maximum gain in the *E*-plane and *H*-plane of a pyramidal horn antenna were given as 0.26λ and 0.4λ , respectively. Our results obtained using eqs. (7) and (9) agree very well with these values. To our knowledge, the improved formulas (7) and (9) have not appeared in the literature before.

Figure 4. Normalized *H*-plane gain of pyramidal horn as a function of A_1 for different lengths of L_2 . [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Figure 5. A_1 versus L_2 obtained using eqs. (2) and (9) compared to values obtained from (3). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Now, an optimization problem is introduced in which the feeding waveguide dimensions (A and B), the frequency (f), and the desired optimum gain (G) are given as input parameters, while equation (3) is used to find the dimension B_1 . The parameters L_1 , L_2 , and A_1 are given by the following equations:

$$L_1 = \frac{B_1^2}{2.1060\lambda} \quad (10)$$

$$L_2 = \frac{A_1^2}{3.1831\lambda} \quad (11)$$

$$A_1 = \frac{A + \sqrt{A^2 - 6.0458(BB_1 - B_1)}}{2} \quad (12)$$

Equation (12) is obtained by enforcing the fact that; for a pyramidal horn to be physically realizable P_E and P_H must be equal. From the geometry of the horn in Figure 1, the following relations for P_E and P_H can be easily derived:

$$P_E = \frac{L_1(B_1 - B)}{B_1} \quad (13)$$

$$P_H = \frac{L_2(A_1 - A)}{A_1} \quad (14)$$

So, our optimization problem here is a one-dimensional problem. The particle swarm optimization (PSO) method is chosen to solve the problem since; recently, we have been interested in the application of PSO in different electromagnetics and microwave circuits problems [12, 13]. The PSO technique has been successfully applied to antenna design [8, 10,

11], and the results proved that this method is powerful and effective for optimization problems. PSO is similar in some ways to genetic algorithms, but requires less computational bookkeeping and generally fewer lines of code, including the fact that the basic algorithm is very easy to understand and implement. The interested reader can refer to [8, 10–13], and the references therein, for details of the PSO algorithm. It should be noted that our application of the PSO differs completely from that presented in [8]. In [8], a closed form expression, which involves 11 constants, for the wider dimension A_1 was derived using the PSO; while in our approach the gain formula (3) is directly optimized using the PSO.

In our implementation of the PSO, B_1 that gives the optimum gain is found using the following fitness function:

$$\text{Fitness} = \text{Equation}(3) - G_{\text{desired}} \quad (15)$$

III. RESULTS

Table I shows 15 different horn design examples, which have been used to compare our PSO results to those obtained using the previous design methods [1–8].

First, to check the validity of our design approach and to compare with the results presented in [6] (since they are the most accurate among [1–8]), eq. (3) has been solved using PSO while using the approximate formulas (1) and (2) instead of the improved formulas (7) and (9). This is done so that we have a fair comparison since (1) and (2) were used in [6]. Specifically, the following equations for L_1 , L_2 , and A_1 are used instead of equations (10)–(12):

$$L_1 = \frac{B_1^2}{2\lambda} \quad (16)$$

$$L_2 = \frac{A_1^2}{3\lambda} \quad (17)$$

$$A_1 = \frac{A + \sqrt{A^2 - 6(BB_1 - B_1)}}{2} \quad (18)$$

Our results for the wider dimension B_1 along with those reported in [6] are shown in Table II. In [6], a neural network was trained to find the value of the dimension B_1 , while it is found directly here using PSO. The desired values for B_1 are also taken

TABLE I. Values of Frequencies and Inner Dimensions of Common Standard Rectangular Feed Waveguides and Desired Gains

Example No.	Waveguide Standard		Frequency (GHz) f	Desired Gain (dB) G	Waveguide Dimensions (cm)	
	I EC	WR			A	B
1	R14	WR650	1.431	15.85	16.51	8.255
2	R22	WR430	2.163	16.50	10.95	5.461
3	R26	WR340	2.736	17.40	8.636	4.318
4	R32	WR284	3.275	18.03	7.214	3.404
5	R40	WR229	4.061	18.80	5.817	2.908
6	R48	WR187	4.968	19.95	4.755	2.215
7	R58	WR159	5.849	20.85	4.039	2.014
8	R70	WR137	6.779	21.75	3.485	1.580
9	R84	WR112	8.290	22.50	2.850	1.262
10	R100	WR90	10.34	22.70	2.286	1.016
11	R120	WR75	12.40	23.20	1.905	0.953
12	R140	WR62	14.95	23.50	1.580	0.790
13	R180	WR51	18.24	23.85	1.295	0.648
14	R280	WR34	27.36	24.30	0.864	0.432
15	R320	WR28	33.22	24.60	0.711	0.356

from [6]. According to [15], these desired values were first obtained roughly using previously published formulas [3–5], then the desired B_1 values, which lead to an excellent agreement between the computed gains of the designed pyramidal horns and the desired gains, were obtained by using an iterative technique. It can be seen that the PSO results are in excellent agreement with the desired B_1 values.

Having verified the design approach, the improved formulas (7) and (9) are now used to find

the wider dimension B_1 using PSO. Our improved results for A_1 and B_1 are given in Table III along with our results from Table II. It can be seen that there are small differences between the two sets of results. The difference ranges from 2 mm for the highest gain example to around 1.5 cm for the lowest gain example. We believe that these small differences are very important since exact dimensions are usually needed for horn antennas when used as a standard gain antenna to determine the precise gain of other antennas.

TABLE II. A Comparison of the Desired B_1 Values with the Results Reported in [6] and Those Obtained Using the PSO with eqs. (1) and (2)

No.	Desired B_1 (m)	Guney [6]		Our Results Using PSO	
		B_1 (m)	Absolute Error	B_1 (m)	Absolute Error
1	0.4386721353106108	0.43867213410993439	1.2007×10^{-9}	0.43867213531014	$4.708455847435289 \times 10^{-13}$
2	0.3144154246448677	0.31441542405646650	5.8840×10^{-10}	0.31441542464479	$7.771561172376096 \times 10^{-14}$
3	0.2774569102970922	0.27745690986449739	4.3259×10^{-10}	0.27745691029704	$5.218048215738236 \times 10^{-14}$
4	0.2496681412209338	0.24966901518942222	8.7397×10^{-7}	0.24966814122088	$5.107025913275720 \times 10^{-14}$
5	0.2214031095703027	0.22140310964312218	7.2819×10^{-11}	0.22140310957023	$6.772360450213455 \times 10^{-14}$
6	0.2072801174889028	0.20728011787642514	3.8752×10^{-10}	0.20728011748883	$7.444045380111675 \times 10^{-14}$
7	0.1963432517473291	0.19634325240751146	6.6018×10^{-10}	0.19634325174725	$7.463474283042615 \times 10^{-14}$
8	0.1880599185382076	0.18805991932393937	7.8573×10^{-10}	0.18805991853804	$1.679489880501706 \times 10^{-13}$
9	0.1679716957223386	0.16797212771119607	4.3199×10^{-7}	0.16797169572227	$7.233103005432895 \times 10^{-14}$
10	0.1378958142667574	0.13789581544039639	1.1736×10^{-9}	0.13789581426670	$5.698219673888616 \times 10^{-14}$
11	0.1222429966826040	0.12224299802912712	1.3465×10^{-9}	0.12224299668255	$5.150047055479945 \times 10^{-14}$
12	0.1050296530375660	0.10502965396363224	9.2607×10^{-10}	0.10502965303752	$4.475586568020162 \times 10^{-14}$
13	0.0896947535384475	0.08969475492360850	1.3855×10^{-9}	0.08969475353841	$3.851086116668512 \times 10^{-14}$
14	0.0630356158868642	0.06303561728956132	1.4027×10^{-9}	0.06303561588684	$2.735311976920229 \times 10^{-14}$
15	0.0537725214751637	0.05377252230591544	8.3075×10^{-10}	0.05377252147514	$2.341876692568690 \times 10^{-14}$

TABLE III. The Aperture Dimensions (in meters) for the 15 Experiments Given in Table I

No	PSO with Improved Formulas		PSO with Formulas (1) and (2)	
	Aperture Dimension A_1	Aperture Dimension B_1	Aperture Dimension A_1	Aperture Dimension B_1
1	0.58745939957790	0.44853806855461	0.57361597193408	0.43867213531014
2	0.41897752185742	0.32160260813165	0.40904904300191	0.31441542464479
3	0.36748291047856	0.28391860822711	0.35840552867341	0.27745691029704
4	0.33732363443676	0.26089097456624	0.32252114892473	0.24966814122088
5	0.29088895881357	0.22667311271466	0.28348102783180	0.22140310957023
6	0.27191636671072	0.21228861617687	0.26486821720179	0.20728011748883
7	0.25560936927822	0.20111475647621	0.24889199855866	0.19634325174725
8	0.24504175271475	0.19266814169802	0.23855032644736	0.18805991853804
9	0.20747207152755	0.16317793605405	0.21260597834625	0.16797169572227
10	0.17916414178191	0.14129224072333	0.17437783238506	0.13789581426670
11	0.15784776775620	0.12525307711374	0.15360236582933	0.12224299668255
12	0.13550390047196	0.10761884737834	0.13185422799275	0.10502965303752
13	0.11560862074174	0.09190862517665	0.11248356493265	0.08969475353841
14	0.08115133942015	0.06459365453271	0.07895527928804	0.06303561588684
15	0.06917393039352	0.05510271231507	0.06729446076815	0.05377252147514

Using the dimensions in Table III, the gain is computed using eq. (3) and the improved formulas (7) and (9). The results are shown in Table IV. It can be seen that there is a difference between the desired and the computed gains when using the aperture dimensions obtained using eqs. (1) and (2). Using the improved aperture dimensions A_1 and B_1 , the desired gain is obtained exactly.

TABLE IV. Computed Gains Using (3) Along with (7) and (9)

No.	Gain Computed Using the Aperture Dimensions Obtained Using (1) and (2)	Gain Computed Using the Improved Aperture Dimensions	Desired Gain
1	15.65225377151011	15.85000000000000	15.85
2	16.30503868563279	16.50000000000000	16.50
3	17.19774296438053	17.40000000000000	17.40
4	17.82432108495824	18.03000000000000	18.03
5	18.59070601292597	18.80000000000000	18.80
6	19.73616544011127	19.95000000000000	19.95
7	20.63332003114509	20.85000000000000	20.85
8	21.53096300646700	21.75000000000000	21.75
9	22.27939335994748	22.50000000000000	22.50
10	22.47893636158694	22.70000000000000	22.70
11	22.97793385306501	23.20000000000000	23.20
12	23.27751657819921	23.50000000000000	23.50
13	23.62681934326002	23.85000000000000	23.85
14	24.07637655171835	24.30000000000000	24.30
15	24.37570568796014	24.60000000000000	24.60

IV. CONCLUSIONS

New improved formulas for the design of optimum gain horn antennas have been proposed. These formulas do not approximate the path length error, and thus, they give more accurate horn dimensions for a given gain. The particle swarm optimization technique has been used to find the wider dimension directly from the exact gain formula. Better accuracy with respect to previous design methods has been obtained for various design examples.

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