A Theoretical and Experimental Study of Coplanar Waveguide Shunt Stubs

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Abstract—A comprehensive theoretical and experimental study of straight and bent coplanar waveguide (CPW) shunt stubs is presented. In the theoretical analysis, the CPW is assumed to be inside a cavity, while the experiments are performed on open structures. For the analysis of CPW discontinuities with air-bridges, a hybrid technique has been developed which has been validated through extensive theoretical and experimental comparisons. Throughout this study, the effect of the cavity resonances on the behavior of the stubs with and without air-bridges is investigated. In addition, the encountered radiation loss due to the discontinuities is evaluated experimentally.

I. INTRODUCTION

COPLANAR WAVEGUIDE (CPW) is rapidly becoming the transmission line of choice in high frequency applications and is successfully competing against the microstrip which has been the primary structure for hybrid and monolithic circuits. Due to many years of microstrip use, a large body of published data and CAD software pertaining to low- and high-frequency microstrip circuit and antenna design has been widely available. On the contrary, models for shielded or open coplanar waveguide circuit design are still under development [1]–[22]. Nevertheless, despite this scarcity of reliable circuit models, CPW has provided an attractive alternative to conventional microstrip lines at high frequencies due to many appealing properties [23]–[31].

While there is no need for via holes in CPW circuits, air-bridges are fundamental components required to connect the ground planes for suppression of the coupled slotline mode. In the past few years, there has been several attempts to characterize, theoretically and/or experimentally, CPW discontinuities with air-bridges or bond wires [16]–[22]. The full wave computationally intensive Finite Difference Time Domain technique was used in [17], [21] to analyze a CPW shunt stub with bond wires and a modified CPW air-bridge T-junction. On the other hand, the Finite Difference Frequency Domain method was used in [22] to treat two common types of CPW air-bridges where it was found that the reflection coefficient (or $S_{11}$) varies linearly with frequency. This fact suggests that a typical air-bridge can be modeled as a frequency dependent lumped element [16], [18], [19], [28], [32]. With this in mind, a hybrid technique has been developed to analyze CPW discontinuities with air-bridges. In this technique, the frequency dependent equivalent circuit of the discontinuity, with the air-bridges removed, is derived using the Space Domain Integral Equation (SDIE) method [8], [9]. Then, this equivalent circuit is modified by incorporating the air-bridge parasitic inductance and capacitance which are evaluated using a quasistatic model.

In this paper, the above mentioned hybrid technique is used to study a variety of CPW shunt stub geometries (Fig. 2) and the validity of the model is verified by performing extensive measurements. The scattering parameters of the stubs with and without air-bridges are presented and a very good agreement is found between theoretical and experimental data. In addition, since a shielded structure is assumed in the formulation of the theory, the effect of cavity resonances on the behavior of these stubs is shown. Moreover, the encountered radiation loss in open discontinuities is investigated experimentally.

II. THEORY

In the theoretical analysis, the CPW under consideration is assumed to be shielded by a rectangular cavity with perfectly conducting walls as shown in Fig. 1. As pointed out in the introduction, a hybrid technique is used to analyze coplanar waveguide discontinuities with air-bridges. First, the frequency dependent equivalent circuit of the discontinuity, with the air-bridges removed, is derived using the Space Domain Integral Equation (SDIE) method [8], [9]. Then, this equivalent circuit is modified by incorporating the parasitic reactances introduced by the air-bridges. Only the main steps of the SDIE method are given here, since the details can be found in [8], [9].
A. SDIE Method

The boundary problem pertinent to any CPW discontinuity may be split into two simpler ones by applying the equivalence principle which introduces an equivalent magnetic current \( \mathcal{M}_s \) on the slot apertures \([8,9]\). This surface magnetic current radiates an electromagnetic field in the two waveguide regions (above and below the slots) so that the continuity of the tangential electric field on the surface of the slots is satisfied. The remaining boundary condition to be applied is the continuity of the tangential magnetic field on the surface of the slot apertures which leads to the following integral equation

\[
\mathcal{I} \times \int_S \left[ \mathcal{K} \left( \mathbf{r}/\mathbf{r}' \right) + \mathcal{K}_1 \left( \mathbf{r}/\mathbf{r}' \right) \right] : \mathbf{M}_s(\mathbf{r}')ds' = \mathbf{J}_s \quad (1)
\]

with \( \mathcal{K} \) and \( \mathcal{K}_1 \) the magnetic field dyadic Green's functions in the two waveguide regions \([8,9]\) and \( \mathcal{I} \) an assumed ideal electric current source exciting the coplanar waveguide mode. The integral equation which is formulated here in terms of equivalent magnetic currents flowing on the slot apertures is different from the fullwave technique presented in \([11,5,14]\), where an integral equation in terms of the electric current on the conducting surfaces is formed. The former technique is more appropriate for CPW problems where the ground planes approach the boundary surfaces (open or shielded), while the latter better fits problems with finite size ground conductors.

The integral equation (1) is solved using the Method of Moments \([33]\) where the unknown magnetic current is expanded in terms of rooftop basis functions. Then, Galerkin’s method is applied to reduce the above equation to a linear system of equations

\[
\begin{pmatrix}
Y_{yy} & Y_{yz} \\
Y_{zy} & Y_{zz}
\end{pmatrix}
\begin{pmatrix}
V_y \\
V_z
\end{pmatrix}
=
\begin{pmatrix}
I_x \\
I_y
\end{pmatrix}.
\quad (2)
\]

In the above, \( Y_{ij}(i = y, z; j = y, z) \) represent blocks of the admittance matrix, \( V_i \) is the vector of unknown \( y \) and \( z \) magnetic current amplitudes, and \( I_j \) is the excitation vector which is identically zero everywhere except at the position of the sources. The solution of the matrix equation (2) results in the evaluation of the equivalent magnetic currents and consequently the electric fields on the slots.

In case of isolated CPW structures which are symmetric with respect to the center line, the coupled slotline mode is not excited, and hence, the aperture fields in the feeding lines form standing waves of the fundamental coplanar waveguide mode. Consequently, using the derived electric field, transmission line theory can be utilized to determine the scattering parameters and derive a lumped equivalent circuit for the discontinuity. In case of asymmetric CPW discontinuities, the derived field in the feeding lines is the sum of the fundamental coplanar and slotline modes each one having its own spatial parameters, and consequently, a special treatment is needed to separate the two modes and derive the scattering parameters of the discontinuity \([35]\).

B. Modeling the Air-Bridges

Fig. 3(a) shows the equivalent circuit (\( \pi \)-model) for the CPW discontinuities shown in Fig. 2 with the air-bridges removed. \( X_1 \) and \( X_2 \) represent the reactances due to the coplanar waveguide mode excited in the CPW stub, while \( X_3 \) is due to the coupled slotline mode \([34]\). The reference planes at which these reactances are evaluated are at the ends of the uniform feeding lines. It should be noted that in the case of straight stubs, the two reactances \( X_1 \) and \( X_2 \) are equal due to the symmetry of the circuit. Thus, only two independent excitations are needed to evaluate the elements of the equivalent circuit, as opposed to three required in the case of the bent stubs \( X_1 \neq X_2 \) \([9]\).

Fig. 3(b) shows the new equivalent circuit after taking the air-bridges into consideration. The air-bridges can be modeled as sections of an air-filled microstrip line \([16,28]\), and simple formulas can be used to evaluate the parasitic capacitance...
Fig. 3. (a) Equivalent circuit of the CPW discontinuities shown in Fig. 2 without air-bridges. (b) The modified equivalent circuit after taking the air-bridges into consideration.

$C_a$ and inductance $L_a$ [34]. Alternatively, a parallel plate waveguide model can be employed to evaluate the same parasitic effects. It has been found that the difference in the values of the parasitic reactances as predicted by the two models has a negligible effect on the performance of the circuit. Finally, new scattering parameters are evaluated from the modified equivalent circuit. It should be emphasized that such a hybrid technique assumes that the air-bridges are positioned as close to the cross junction as possible, which is always the case in practice. Moreover, it is worth mentioning that the CPU time required for the evaluation of the scattering parameters (taking into account all the existing physical symmetries) ranges from one to three minutes for each frequency on Apollo Domain 10 000.

III. RESULTS AND DISCUSSION

In the numerical results shown here, the considered CPW discontinuities are suspended inside a rectangular cavity, as shown in Fig. 1, with $h = 400 \mu m$, $\varepsilon_{r1} = 13$, $\varepsilon_{r2} = 1$, $S = 75 \mu m$, $W = 50 \mu m$, and $h_1 = h_2 = 1.2 \ mm$. The characteristic impedance of such a line is approximately 50$\Omega$.

On the other hand, the slots and center conductor of the CPW stubs have equal widths of 25$\mu$m. In addition, in the case of the open-end stubs, the width of the open-end is 25$\mu$m. In all examples presented here, the stubs are placed symmetrically at the center of the cavity with length approximately equal to $3\lambda_0$.

The experiments were performed in an open environment with the CPW circuits fabricated on 400$\mu$m thick GaAs using lift-off processing. The CPW center strip and ground planes consist of 200$\ \AA$ of Cr and 1.5$\mu$m of Au. The air-bridges have 10$\mu$m square posts and are 14$\mu$m wide. The air-bridge thickness and height are 1.0$\mu$m and 3.0$\mu$m, respectively. The GaAs circuit rests on a piece of 3.175 mm 5880 RT/durid which has a dielectric constant of 2.2.

It will be shown that the theoretical and experimental data agree despite the difference in $\varepsilon_{r2}$ of the substrate ($\varepsilon_{r2} = 1$ versus 2.2). This is due to the relatively large thickness of the GaAs substrate layer as compared to $S + 2W$. The choice of $\varepsilon_{r2} = 1$ in the theoretical calculations is intended in order to avoid some of the unwanted cavity resonances. The circuits were tested with an HP 8510 network analyzer and a Cascade probe station. The calibration standards for a TRL calibration were fabricated on the wafer to allow calibration to the reference planes of the CPW stubs. To cover the 5–40 GHz bandwidth, three delay lines were used. The probe positioning on the wafer was determined to be repeatable to within 3$\mu$m which creates a maximum error in the phase of $\theta_{21}$ of 0.76° at 40 GHz.

A. Open-End Shunt Stubs

Fig. 4 shows the magnitude of the scattering parameters of the straight open-end stub (Fig. 2(a)) of length $L = 1100\mu$m with the air-bridges removed as a function of frequency. It is noticed that the theoretical and experimental results agree very well up to the first resonance, after which, discrepancy is noticeable. This is attributed to the fact that no loss is assumed in the theoretical formulation, while, experiments were performed on CPW structures in an open environment. The loss encountered in the measurements is mainly due to radiation by the slotline mode excited in the CPW stubs in the absence of the air-bridges. To understand the unexpected behavior seen between 41 and 45 GHz, the reactances $X_1$ and $X_3$ of the equivalent circuit are plotted in Fig. 5. It can be seen that $X_1$, which corresponds to the coplanar mode in the CPW stub, behaves as a real open-end shunt stub. On the other hand, $X_3$, which is due to the slotline mode in the CPW stubs, behaves somewhat anomalously between 41
and 45 GHz. It is found that this behavior is due to a cavity resonance at 43 GHz which corresponds to the $LSM_{121}$ mode excited in the partially filled lower cavity (the cavity width was chosen to be 3.425 mm). It is interesting to note that the $LSM_{111}$ cavity mode, which has a resonant frequency of 38 GHz, does not show an observable effect on the stub under consideration. Also, the shunt reactances seem to be unaffected by the $LSM_{121}$ cavity mode.

Fig. 6 shows the scattering parameters (magnitude and phase) of the same straight open-end stub with air-bridges. The good agreement between the theoretical and experimental results validates the developed hybrid technique. As it can be seen, the stub resonates at nearly 24.5 GHz where the length of the stub is approximately quarter of a coplanar mode wavelength. It can be noticed that the agreement between the theoretical and experimental data in Fig. 6(a) is much better than the one seen in Fig. 4. This is due to the fact that the air-bridges tend to prevent the slotline mode from being excited in the CPW stubs which effectively reduces radiation losses encountered in the measurements. In addition, it can be seen that the resonance effect noticed in Fig. 4 has disappeared since $X_3$ is shorted by the relatively small air-bridge inductance $L_{ab}$. As a result, with the presence of air-bridges, cavity resonances have no effect on the characteristics of a straight stub as long as it is placed symmetrically inside the cavity. One can also notice an anomalous effect at 40 GHz existing in Fig. 6. A similar effect has been found in [17] for the case of a bent open-end stub. This effect may be due to a resonating slotline mode excited in the stubs beyond the air-bridge.

Fig. 7 shows the magnitude of the scattering parameters for the bent open-end CPW stub (Fig. 2(b)) of mean length 1100 $\mu$m without air-bridges. In this case, the width of the cavity is taken to be 2 mm. It can be seen that the agreement between theory and experiment is better than that seen in Fig. 4 for the straight stub without air-bridges. Moreover, it has been found that the parallel combination of $X_1$ and $X_2$ behaves as expected for a real shunt open-end stub (i.e. similar to the variation of $X_1$ in Fig. 5). Fig. 8 shows the magnitude of the scattering parameters of the same bent open-end stub with air-bridges. It can be seen that the anomalous effect at 40 GHz is more pronounced in this case than that in Fig. 6. In addition, the resonant frequency of this bent stub with air-bridges is approximately the same as the one for the straight stub of the same mean length.

### B. Short-End Shunt Stubs

Figs. 9 and 10 show $Mag(S_{11})$ for the straight and bent short-end CPW stubs, respectively, of mean length $L = 1100 \mu$m with and without air-bridges. It can be noticed that both stubs resonate at approximately 25.5 GHz. The same arguments presented above for the open-end stub hold here too.
It is interesting to note that for all structures containing air-bridges, the measured resonant frequency is larger than the theoretically predicted one. This systematic deviation may be due to the effect of finite metallization thickness (1.5 μm) which is neglected in the theoretical analysis. The finite metallization thickness reduces the phase constant [36] and thus increases the stub resonant frequency. EEsof Touchstone has predicted an increase of approximately 1.2% in the resonant frequency of a section of CPW of length 1100 μm due to the finite conductor thickness.

Since the experiments were performed on open CPW discontinuities, the measured scattering parameters can provide the loss factor. This loss factor includes radiation, conductor and dielectric losses. However, since the stubs are all of the same length and printed on the same substrate, a comparison of the loss factor can provide a measure of the radiation loss. Figs. 11(a) and 12(a) show the loss factor of the short-end and open-end stubs without air-bridges, respectively. It is noticed that loss is maximum at the resonant frequency for all stubs, which is similar to what has been found in microstrip stubs [37]. Furthermore, the loss factor for straight stubs is larger than that for bent stubs especially after resonance. This behavior is due to the fact that in the case of bent stubs, the fields radiated by the coupled slotline mode in the two opposing stubs.
Fig. 11. The measured loss factor \(1 - |S_{11}|^2 - |S_{12}|^2\) of the short-end CPW stubs. (a) Without air-bridges. (b) With air-bridges.

Fig. 12. The measured loss factor of the open-end CPW stubs. (a) Without air-bridges. (b) With air-bridges.

partially cancel. This explains why the agreement between the theoretical and experimental results for the bent stubs without air-bridges (Fig. 7 and Fig. 10) is better than that for the straight stubs without air-bridges (Figs. 4 and 9). The loss factor for the stubs with air-bridges is shown in Figs. 11(b) and 12(b). It can be seen that the presence of air-bridges reduces the loss factor appreciably since they short out the radiating coupled slotline mode. However, it is still noted that the straight stubs have increasing radiation loss after the first resonant frequency. Finally, it can be noticed that the difference between the loss factor of the straight and bent stubs (with or without air-bridges) below the resonant frequency is within the experimental error. Thus, no concluding remarks can be made concerning the loss from the two stubs in this region.

IV. CONCLUSIONS

A comprehensive theoretical and experimental study of CPW shunt stubs has been presented. In the theoretical analysis, the CPW was assumed to be inside a cavity, while, the experiments were performed in an open environment. A hybrid technique has been developed to analyze the CPW discontinuities which proved valid since the results obtained theoretically and experimentally agreed very well. It was found that bent stubs without air-bridges tend to radiate less than straight stubs of the same length. In addition, the effect of the cavity resonances on the behavior of the stubs has been studied. It has been shown through experiments that bent CPW stubs should be used whenever the circuit layout permits to reduce the radiation loss caused by the parasitic coupled slotline mode.

REFERENCES


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